Optimization of Scheduling for Appointment Based Healthcare Delivery Systems

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Health Systems Engineering

- Simulation
- Optimization
- Decision Analysis
- Quality Control

Industrial & Systems Engineering

- Health Science
- HSE
- Computer Science

Medicine and Health Policy
Operations Management
Hospital Logistics
Decision Support Systems
Health Care Management Science

- Simulation
- Optimization
- Decision Analysis
- Quality Control
- Management Science
- HCMS
- Health Science
- Computer Science
- Decision Support Systems
- Hospital Logistics
- Operations Management
- Medicine and Health Policy
NCSU Industrial & Systems Engineering

- 4\textsuperscript{th} Largest Industrial & Systems Engineering department in the U.S.

- Ranked 13\textsuperscript{th} by U.S. News & World Report

- Undergraduate, Masters and PhD programs

- In 2006, a self-study identified Healthcare as a “focus area”
Health Systems Faculty

Steve Roberts

Javad Taheri

Julie Ivy

Brian Denton

+ Many other faculty specializing in biomanufacturing, human factors engineering, and other areas
Application Interests

Breast Cancer, Primary Care, Disease Screening, Surgery, Healthcare Delivery, Oncology, Emergency Response, Diabetes, Medical Decision Making, Hospital Management, Chronic Disease, Operations Management, Treatment Policies, Endoscopy, Medical Imaging, Colorectal Cancer, Heart Disease, Medication, Prostate Cancer.
Methodological Interests

- Queuing
- Decision Analysis
- Linear Programming
- Operations Research
- Graph Theory
- Discrete Optimization
- Mathematical Programming
- Stochastic Programming
- Markov Decision Processes
- Dynamic Programming
- Six Sigma
- Statistical Process Control
- Simulation
- Statistics
Summary

• Surgery process and complicating factors

• 3 Problems:
  • Single OR scheduling
  • Multi OR planning and surgery allocation
  • Scheduling of an Outpatient Procedure Center

• Future research
Collaborators

Hari Balasubramanian (University of Massachusetts)
Sakine Batun (University of Pittsburgh)
  Bjorn Berg (NCSU)
  Ayca Erdogan (NCSU)
  Todd Huschka (Mayo)
Andrew Miller (University of Bordeaux)
  Heidi Nelson (Mayo)
  Ahmed Rahman (Mayo)
Andrew Schaefer (University of Pittsburgh)

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Surgery Process

- **Patient Intake**: administrative activities, pre-surgery exam, gowning, site prep, anesthetic

- **Surgery**: incision, one or multiple procedures, pathology, closing

- **Recovery**: post anesthesia care unit (PACU), ICU, hospital bed
Outpatient Procedure Centers

- Daily OR and staff planning
- Surgery-to-OR assignments
- Staff scheduling
- Patient arrival time scheduling
- Planning equipment resources (surgical kits, diagnostic equipment)
Complicating Factors

- Many types of resources to be scheduled: OR team, patients, equipment, materials
- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Many competing criteria
Surgery Duration Uncertainty

Minutes

ADND
Problem 1: Single OR Scheduling
Single OR Scheduling - S(n)/G(n)/1

Planned OR Time

Min\{ Idling + Waiting + Overtime\}
Stochastic Optimization Model

\[ \min \left\{ \sum_{i=1}^{n} C_i^w \cdot E_Z[W_i] + \sum_{i=1}^{n} C_i^s \cdot E_Z[S_i] + C_L \cdot E_Z[L] \right\} \]

\[ W_i = \max (W_{i-1} + Z_{i-1} - x_{i-1}, 0) \]

\[ S_i = \max (-W_{i-1} - Z_{i-1} + x_{i-1}, 0) \]

\[ L = \max (W_n + Z_n + \sum x_i - d, 0) \]
Literature Review – Single Server

• Queuing Analysis:
  ▪ Mercer (1960, 1973)
  ▪ Jansson (1966)
  ▪ Brahimi and Worthington (1991)

• Heuristics:
  ▪ White and Pike (1964)
  ▪ Soriano (1966)
  ▪ Ho and Lau (1992)

• Optimization:
  ▪ Weiss (1990) – 2 surgery news vendor model
Stochastic Linear Program

\[
\min \left\{ E_Z \left[ \sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l \right] \right\}
\]

s.t. \quad w_2 - s_2 = Z_1 - x_1

\begin{align*}
- w_2 + w_3 - s_3 &= Z_2 - x_2 \\
- w_n - s_n + l - g &= Z_n - d + \sum_{j=1}^{n-1} x_i \\
x_i &\geq 0, w_i \geq 0, s_i \geq 0, i = 1, \ldots, n, \quad l, g \geq 0
\end{align*}
Two Stage Recourse Problem

Initial Decision ($x$) $\rightarrow$ Uncertainty Resolved $\rightarrow$ Recourse ($y$)

$$\min \{ Q(x) = E_Z[Q(x, Z)] \}$$

$$Q(x, Z^k) = \min \{ c \cdot y^k \mid T x + W y^k = h^k, y^k \geq 0 \}$$
Example

• Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with U(1,2):
Problem 2: Multi-OR Surgery Allocation
Multi-Operating Room Scheduling

Decisions:
• How many operating rooms (ORs) to open?
• Which OR to schedule each surgery block in?

Performance Measures:
• Cost of operating rooms opened
• Overtime costs for operating rooms
Extensible Bin Packing

\[ x_j = \begin{cases} 
1 & \text{if OR } j \text{ open} \\
0 & \text{if OR } j \text{ closed}
\end{cases} \]

\[ y_{ij} = \begin{cases} 
1 & \text{if Surg. Block } i \text{ assigned to OR } j \\
0 & \text{Otherwise}
\end{cases} \]

\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + c^v o_j \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} \bar{z}_i y_{ij} - o_j \leq d_j x_j \quad \forall (i, j) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j \geq 0 \]
Symmetry

• m! optimal solutions:

• Anti-symmetry constraints:

\[ x_1 \geq x_2 \]
\[ x_2 \geq x_3 \]
\[ \vdots \]
\[ x_m \geq x_{m-1} \]

OR Ordering

\[ \sum_{j=1}^{m} y_{mj} = 1 \]

Surgery Assignment

\[ y_{11} = 1 \]
\[ y_{21} + y_{22} = 1 \]
\[ \vdots \]
\[ y_{11} + y_{m1} = 1 \]
Two-Stage Stochastic MIP

\[ Q(\mathbf{x}) = \min\left\{ \sum_{j=1}^{m} c^f x_j + c^v E_\omega[o_j(\omega)] \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]
\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]
\[ \sum_{i=1}^{n} z_i(\omega) y_{ij} - o_j(\omega) \leq d x_j \quad \forall (i, j, \omega) \]
\[ y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \geq 0, \forall \omega \]
Integer L-Shaped Method

Master Problem:
\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + \Theta \right\} \]
\[ s.t. \quad y_{ij} \leq x_j \quad \forall (i, j) \]
\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]
\[ y_{ij}, x_j \in \{0, 1\}, \Theta \geq 0 \]
Heuristic

Longest processing time first heuristic:

EBP Heuristic:

\[ n \leftarrow LB; \]
\[ \text{repeat}; \]
\[ LPT(n); \]
\[ \text{if } (o_j = 0, \forall j) \text{ Stop}; \]
\[ n \leftarrow n + 1; \]
\[ \text{end(repeat);} \]

\[ LB = \left[ \sum_{i=1}^{n} \bar{z}_i \right] \left[ T(1 + \frac{c^f}{c^vT}) \right] \]

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room
Robust Formulation

\[ Z = \min \left\{ \sum_{j=1}^{m} c_f^i x_j + F(x, y) \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ y_{ij}, x_j \in \{0,1\} \geq 0 \]

\[ F(x, y) = \begin{cases} 
\max_{\delta} \left\{ \sum_{j=1}^{m} \eta_j \right\} \\
\text{s.t.} \quad \eta_j = c^i_j \max \left\{ 0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - T_j x_j \right\}, \quad \forall j \\
\quad \sum_{(i,j):y_{ij}=1} \frac{\delta_{ij} - z_i}{z_i - z_i} y_{ij} \leq \tau \\
\quad z_i \leq \delta_{ij} \leq \bar{z}_i, \forall (i, j) : y_{ij} = 1 
\end{cases} \]
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<th>MV_IP</th>
<th>LPT_Heu</th>
<th>Tau=2</th>
<th>Tau=4</th>
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<th>MV_IP</th>
<th>LPT_Heu</th>
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</table>

15 surgery instances

Variable Cost = 0.033

Variable Cost = 0.0083

Robust IP

Robust IP
General Insights

• The LPT based heuristic works (fairly) well on a large number of instances
  – LPT works very well when overtime costs are low
  – LPT is better (and easier) than solving MV problem in most cases
• Robust IP is better than LPT when overtime costs are high

Current Research: Sharing ORs

Problem 3: Patient Arrival Scheduling
Background

- Endoscopy practice at Mayo Clinic
- 4 endoscopists sharing 8 procedure rooms
- Approximately 45 patients per day
- Day begins at 7:30 AM and finishes at 5:00 PM

Endoscopy Suite

Patient Check-in

Waiting Area

Preoperative Waiting Area

Procedure Rooms

Recovery Area

Patient Waiting Time

Patient Arrivals

Schedule

Intake Area

Length of Day

1st Patient Arrival

nth Patient Completion

Patient Discharge
Performance Measures

- Patient throughput
- Waiting time:
  - Patients
  - OR Team
- Utilization:
  - Procedure room
  - OR Team
  - Recovery beds
- Overtime
Model Building

- Process Map:
  - Sequence of activities
  - Patterns of resource utilization
  - Decision points

- Conceptual Model

- Quantitative Models
Process Map

**Intake**
- Patient arrives at the hospital lobby
- Patient notifies the lobby front desk that s/he is here
- Patient walks down to a seating area, takes a seat and waits to be called for check-in
- Patient completes the check-in process

**Surgery**
- Patient walks down or taken to the operation room
- Patient may wait to go to the operation room
- Patient receives consultation, dress change instructions and changes dress
- Patient walks down or taken to the dress room

**Surgery**
- Patient is put on the OR bed and waits for the OR team to arrive
- Patient is given IV if needed and monitored
- Surgeon arrives at the OR. Patient gives consent for operation to the surgeon
- Patient is sedated
- Patient is intubated
- Patient is extubated

**Recovery**
- Patient is discharged
- Patient recovers in the recovery area
- Patient is taken to the recovery area
- Patient may wait to go to the recovery area
<table>
<thead>
<tr>
<th>Instance_ID</th>
<th>Location</th>
<th>Room</th>
<th>Proc</th>
<th>Endoscopist</th>
<th>Primary Nurse</th>
<th>Appt Time</th>
<th>Pt. Status</th>
<th>Status time</th>
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</table>
Intake, Surgery, and Recovery
Simulation-Optimization

- **Decision variables:** scheduled start times to be assigned to $n$ patients each day

- **Goal:** Generate the set of non-dominated schedules to understand tradeoffs between waiting and length of day

- **Methods:**
  - Schedules generated using a genetic algorithm (GA)
  - Non-dominated sorting used to identify the Pareto set and feedback into GA
Pareto Set

- Non-dominated sorting genetic algorithm of Deb et al. (2000) used to rank schedules
Selection Procedure

• Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) used to determine if solution $i$ “dominates” $j$

• Solution $i$ “dominates” $j$ if:

\[
E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]
\]
Genetic Algorithm

- **Main features of the GA:**
  - Randomly generated initial population of schedules
  - Selection based on 1) ranks and 2) crowding distance
  - Single point crossover:

![Diagram of single point crossover](image)

- **Mutation**
Appointment Schedules

![Graph showing the relationship between Mean Length of Day and Mean Waiting Time, with points indicating dominated schedules.](image)
Conclusions

• Simple “hedging” heuristic is nearly as good as more complicated genetic algorithm:
  
  – Set initial appointment time to start of day:
    \[
    a_i = 0
    \]
  
  – Set remaining appointment times incrementally
    \[
    a_i = a_{i-1} + \mu + \Delta
    \]
Current and Future Research

• Dynamic (online) scheduling problems

• Study of no-shows and overbooking

• Accounting for rescheduling during the day

• Planning and scheduling of chemotherapy treatment centers
Questions?

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