Stochastic Optimization for Scheduling in Healthcare Delivery Systems

Optimization Days
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Summary

Optimization in Healthcare

Surgery Scheduling Examples:

- Example 1: Single OR scheduling
- Example 2: Multi-OR scheduling
- Example 3: Bi-criteria scheduling of multi-stage surgery suite

Wrap-up
Optimization in Healthcare

Nurse Scheduling

Ambulance Dispatching

Primary Care Panels

Inventory Management
Healthcare in Optimization

# of Health Care Talks at INFORMS Annual Meetings

- San Antonio (2000)
- Miami (2001)
- San Jose (2002)
- Atlanta (2003)
- Denver (2004)
- San Francisco (2005)
- Pittsburgh (2006)
- Seattle (2007)
- Washington DC (2008)
- San Diego (2009)
- Austin (2010)
- Charlotte (2011)
- Phoenix (2012)
Warning: Shameless Advertising

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Surgical Care Delivery

- Efficient access to surgery is important for patient health and safety

- Surgery accounts for the largest proportion a hospital’s expenses and revenues
Surgery in the U.S.

- **Hospitals**
  - Open 24 hours a day
  - Patients recover in the hospital
  - Handle complex surgeries

- **Ambulatory Surgery Centers**
  - Normally open 7am to 5pm
  - Patients admitted and discharged same day
  - Lower cost and lower infection rate than hospitals
Patient Intake: administrative activities, pre-surgery exam, gowning, site prep, anesthetic

Surgery: incision, one or multiple procedures, pathology, closing

Recovery: post anesthesia care unit (PACU)
Blue lines represent patient flow

Pre/post rooms

Operating rooms

Patient waiting area
Management decisions that can be supported with optimization models

- Surgery start time scheduling
- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling of staff in intake, surgery, and recovery
Complicating Factors

- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Multiple competing criteria
Empirical distribution for tonsilectomy
Empirical distribution for hernia repair
Example 1
Single Operating Room (OR) Scheduling
Single OR Scheduling Problem

For a single OR find the optimal time to allocate for each surgery to minimize the cost of:

- Patient and surgery team waiting
- Unutilized (idle) time of the operating room
- Overtime
Single OR Scheduling

Planned OR Time (e.g. 8 hours)

Example Scenario:

Goal: $\text{Min}\{\text{Idling} + \text{Waiting} + \text{Overtime}\}$
Stochastic Optimization Model

Cost of Waiting

\[
\min_x \left\{ \sum_{i=1}^{n} C_i^w E_Z[W_i] + \sum_{i=1}^{n} C^s E_Z[S_i] + C^L E_Z[L] \right\}
\]

Cost of Idling

Random surgery time

\[
W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)
\]

Planned time for surgery

\[
S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)
\]

\[
L = \max(W_n + Z_n + \sum x_i - T, 0)
\]
Literature Review – Single Server

Queuing Analysis:
- Mercer (1960, 1973)
- Jansson (1966)
- Brahimi and Worthington (1991)

Assumes steady state is reached, i.i.d. service times, fix time allotment

Heuristics:
- White and Pike (1964)
- Soriano (1966)
- Ho and Lau (1992)

No guarantee of optimal solution

Optimization:
- Weiss (1990) – 2 surgery news vendor model
- Denton and Gupta (2003) – General stochastic programming formulation
Reformulation as a Stochastic Program

\[
\min \{ E_Z \left[ \sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l \right] \}
\]

s.t. \quad w_2 - s_2 = Z_1 - x_1
\quad - w_2 + w_3 - s_3 = Z_2 - x_2
\quad - w_n - s_n + l - g = Z_n - d + \sum_{j=1}^{n-1} x_i

x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1, \ldots, n, \quad l, g \geq 0
Two Stage Recourse Problem

Initial Decision \((x) \rightarrow \) Uncertainty Resolved \(\rightarrow\) Recourse \((y)\)

\[
\min\{ Q(x) = E_Z[Q(x, Z)] \}
\]

\[
Q(x, Z^k) = \min\{ c \cdot y^k \mid T \cdot x + W \cdot y^k = h^k, y^k \geq 0 \}
\]

Solve using L-shaped method
Example: Surgery allocations for \( n=3, 5, 7 \) patients with i.i.d. \( U(1,2) \)
Insights

- Simple heuristics often perform poorly
- The value of the stochastic solution (VSS) can be high
- Large instances of this problem can be solved very easily


There are many variations on this problem

- No-shows
- Tardy arrivals
- Dynamic scheduling
- Robust formulations
- Endogenous uncertainty


Example 2
Multiple Operating Room Surgery Allocation
Multi-OR Scheduling Problem

Given a set of surgeries to be scheduled on a certain day decide the following:

- How many ORs to make available to complete all surgeries
- Which OR in which to perform each surgery block
Multi-OR Scheduling Problem

Decisions:
- How many ORs to open each day?
- Which OR to schedule each surgery block in?
Extensible Bin-Packing

\[ x_i = \begin{cases} 
1 & \text{if OR i active} \\
0 & \text{otherwise} 
\end{cases} \quad \begin{cases} 
1 & \text{if surgery } j \text{ assigned to OR } i \\
0 & \text{Otherwise} 
\end{cases} \]

\[ Z = \min\{\sum_{i=1}^{m} c^f x_i + c^v o_i\} \]

s.t. \[ y_{ij} \leq x_i \quad i = 1, \ldots, m, \ j = 1, \ldots, n \]
\[ \sum_{i=1}^{m} y_{ij} = 1 \quad j = 1, \ldots, n \]
\[ \sum_{j=1}^{n} d_j y_{ij} - o_i \leq Tx_i \quad i = 1, \ldots, m \]
\[ y_{ij}, x_i \text{ binary}, \ o_i \geq 0 \]

Cost of ORs + Overtime

Surgeries only scheduled in ORs that are active

Every surgery goes in one OR

Overtime if surgery goes past end of day of length \( T \)
Stochastic MIP with random surgery durations

\[ Q(\mathbf{x}) = \min \{ \sum_{j=1}^{m} c^f x_j + c^v E_\omega [o_j(\omega)] \} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} d_i(\omega)y_{ij} - o_j(\omega) \leq T x_j \quad \forall (i, j, \omega) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \geq 0, \forall \omega \]
Symmetry is a problem

There are m! optimal solutions:

Adding the following anti-symmetry constraints reduces computation time:

\[
\begin{align*}
x_1 & \geq x_2 \\
x_2 & \geq x_3 \\
& \vdots \\
x_m & \geq x_{m-1}
\end{align*}
\]

**OR Ordering**

\[
\begin{align*}
y_{11} & = 1 \\
y_{21} + y_{22} & = 1 \\
& \vdots \\
\sum_{j=1}^{m} y_{mj} & = 1
\end{align*}
\]

**Surgery Assignment**
**Integer L-Shaped Method**

Branch and bound tree:

- **IP0**
  - **IP2**
    - **IP4**
    - **IP6**
      - **IP8**
  - **IP1**
    - **IP3**
      - **IP5**
    - **IP7**

**Master Problem:**

\[
Z = \min \left\{ \sum_{j=1}^{m} c^f_x + \Theta \right\}
\]

subject to:

- \( y_{ij} \leq x_j \quad \forall (i, j) \)
- \( \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \)
- \( y_{ij}, x_j \in \{0,1\}, \Theta \geq 0 \)

(optimality cuts)
\[
\Theta \geq E_\omega [\pi (h - Tx)]
\]
Longest Processing Time First Heuristic

Sort surgeries in LPT order;
m ← LB on number of ORs;
while(o_j = 0, ∀j)
    LPT(m);
    m ← m + 1;
end
Compute m^* with lowest total cost

Robust Formulation

Robust formulation seeks to minimize the worst case cost.

\[
Z = \min\left\{ \sum_{j=1}^{m} c^f x_j + F(x, y) \right\}
\]

\[
\text{s.t.} \quad y_{ij} \leq x_j \quad \forall (i, j)
\]

\[
\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)
\]

\[
y_{ij}, x_j \in \{0,1\} \geq 0
\]

Worst case (adversary) problem

\[
F(x, y) = \left\{ \begin{array}{l}
\max_{\delta}\left\{ \sum_{j=1}^{m} \eta_j \right\} \\
\text{s.t.} \quad \eta_j = c^y \max\{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - dx_j\}, \quad \forall j \\
\sum_{(i,j):y_{ij}=1} \frac{\delta_{ij} - z_i}{z_i - z_i} y_{ij} \leq \tau \\
z_i \leq \delta_{ij} \leq z_i, \forall (i,j) : y_{ij} = 1
\end{array} \right.
\]

Uncertainty budget
Results of sample test problems

<table>
<thead>
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<th>Instance</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
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<td>.85</td>
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<tr>
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<td>.92</td>
<td>.90</td>
<td>.97</td>
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Table 1: Cost of 0.5 hours overtime equal cost, \( c_f \), of opening an OR

<table>
<thead>
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<th>1</th>
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<th>4</th>
<th>5</th>
<th>6</th>
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<td>1.0</td>
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<tr>
<td>MV</td>
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<td>.88</td>
<td>.97</td>
<td>.99</td>
<td>.96</td>
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<td>.95</td>
</tr>
</tbody>
</table>

Table 2: Cost of 2 hours overtime equal cost, \( c_f \), of opening an OR

LPT = longest processing time first heuristic, MV = mean value problem, Robust = solution to robust integer program. Results expressed as the ratio of optimal solution to solution generated by MV, LPT, Robust
Insights

• LPT works well when overtime costs are low

• LPT is better (and much easier) than solving MV problem in most cases

• Robust IP is better than LPT when overtime costs are high

Relaxing assumptions about assignment decisions leads to challenging problems.

LPT Heuristic Analysis

Extension to Dell’Ollmo et al. (1998) to consider extensible bins with costs

**Theorem:** The LPT heuristic has the following performance ratio:

\[
\frac{C_{LPT}}{C^*} \leq \frac{Sc^v}{12cf}
\]

and there exist instances where the bound is tight.


Example 3
Patient Arrival Scheduling in Multi-Stage Procedure Center
Patient Arrival Scheduling Problem

Find the Pareto optimal appointment times for patients having a procedure in an ambulatory surgery center to trade-off:

- Expected patient waiting
- Expected length of day
Endoscopy Suite

Patient Check-in Waiting Area

Preoperative Waiting Area

Intake Area

Operating Rooms

Recovery Area

Patient Waiting Time

Schedule

First Patient Arrival

Length of Day

Last Patient Completion

Patient Arrivals

Patient Discharge

Endoscopy Suite
Intake, Procedure and Recovery Distributions
Simulation-optimization

Decision variables: scheduled start times to be assigned to $n$ patients each day

Goal: Generate Pareto optimal schedules to understand tradeoffs between patient waiting and length of day

- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA
Pareto Set

The non-dominated sorting genetic algorithm (NSGA-II) of Deb et al. (2000):
Selection Procedure

Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) to compute the number of samples necessary to determine whether a solution \( i \) “dominates” \( j \)

Solution \( i \) “dominates” \( j \) if:

\[
E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]
\]
Genetic Algorithm

- Randomly generated initial population of schedules
- Selection based on 1) ranks and 2) crowding distance
- Mutation
- Single point crossover:

Parents

\[ z_1 \quad z_2 \quad z_3 \quad \ldots \quad z_n \]
\[ y_1 \quad y_2 \quad y_3 \quad \ldots \quad y_n \]

Children

\[ z_1 \quad z_2 \quad - \quad y_3 \quad \ldots \quad y_n \]
\[ y_1 \quad y_2 \quad - \quad z_3 \quad \ldots \quad z_n \]
Schedule Optimization

Average Length of Day vs. Average Waiting Time

- Efficient Frontier
- Dominated points
Insights

- A simple simulation optimization approach provides significant improvement to schedules used in practice

- Controlling the mix of surgeries each day can improve both patient waiting time and overtime

Many healthcare delivery systems have complex interactions

Key Points

- There are many open opportunities for research in optimization of healthcare delivery systems
- New problems help drive creation of new methods and theory
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