

## Lecture 2 Example: Lotsizing Problem

This short document provides detailed solution to the lotsizing problem discussed in Lecture 2. See lecture slides and in-class notes for more details.

A company knows that demand for a product for the next 4 months is:

Month	Demand
1	1
2	3
3	2
4	4

At the beginning of each month the company decides how many units to produce. During a month in which any units are produced there is a setup cost of \$3. In addition, there is a variable cost of \$1 per unit produced. At the end of each month there is a holding cost of \$0.5 per unit. Determine the optimal production schedule.

The optimality equations for an n period problem are:

$$v_i = \min_{j=i+1, \dots, n} \{c_{ij} + v_j\}, \quad v_n = 0$$

Thanks to the Theorems from Wagner and Whitin discussed in Lecture 3 it is only optimal to produce when there is zero inventory. Therefore each stage has a single state (zero inventory). The optimal action is how many periods of demand to produce for.

Solution to Optimality Equations:

First note that because the per unit production cost is the same in each period we can ignore it in the solution of the problem since it will be a constant ( $\$1 \times \text{Total Demand} = \$10$ ) no matter what the policy is.

$$v_5 = 0 \text{ (boundary condition)}$$

$$v_4 = c_{40} = 7 \text{ (cost to produce for period 4 demand)}$$

$$v_3 = \min\{c_{30} + v_4, c_{31}\} = 11 \text{ (min of cost to produce for period 3 and 4, or just period 3)}$$

$$v_2 = \min\{c_{20} + v_3, c_{21} + v_4, c_{22}\} = 16.5$$

$$v_1 = \min\{c_{10} + v_2, c_{11} + v_3, c_{12} + v_4, c_{13}\} = 19.5$$

Optimal Solution:

Total cost = Total fixed cost + inventory holding cost + production cost = 19.5 = 19.5

Optimal Policy: In the first period produce either for 2 or 3 periods of demand (both options have the same cost, i.e., there are 2 optimal policies). If you produce for 2 period then in period 3 produce for periods 3 and 4 demand. If, on the other hand, you initially produce for 3 periods, then in period 4 produce to cover the demand in period 4.