Last Time

Lot-sizing Problem

- Wagner-Whitin algorithm

Important Points

Office Hours: Tuesday 4-5pm, Thursday 2-3pm.

Assignment 1 due on Thursday at <u>start of class</u>

Assignment 2 up soon

Midterm Exam: Take Home - Starts Oct 23 at 10:30am, Due 10:30am Oct 26)

Today

- Curse of dimensionality
- Equipment replacement problem (ERP)
  - Covered in Chapter 2 of Dreyfus and Law

Limits of DP

In some problems the state space becomes so large that excessive computation time is required to solve the DP

Consider the following examples. What is the “state” for each?

- A system of checkout counters at a grocery store where the state is defined by the number of people in line at each counter.
- A shoe store that stocks many types of shoes where the state is defined by inventory levels
- A transportation system where the state is defined by the number of vehicles in each part of the traffic system

Equipment Replacement

Many companies and customers face the problem of determining how long a piece of equipment should be utilized before replacement

- As equipment ages the cost of maintenance increases
- The trade in value of the equipment decreases over time

Problems of this type are called **equipment replacement problems (ERPs)**

ERP: DP Formulation

States: age of machine at stage t, s_t

Actions: $a_t \in \{Buy, Keep\}$

Rewards:

- $c(s_t) =$ cost of operating a machine of age s_t
- $p =$ price of new machine
- $r(s_t) =$ trade-in value for machine of age s_t

Optimality Equations:

$$v_t(s_t) = \min_{a_t \in \{B,K\}} \{p - r(s_t) + c(0) + v_{t+1}(1), c(s_t) + v_{t+1}(s_t + 1)\}, \quad \forall s_t$$
$$v_T(s_T) = -\mathrm{r}(s_T), \quad \forall s_T$$

Example: Equipment Replacement

Assume a new tool must be purchased at time 0. A new tool costs $12,000. The time horizon is 5 years and the salvage value at the end of year 5 is the trade-in price. Find the optimal policy.

Age of Tool (Years)	Annual Maintenance cost $c(s_t)$	Trade-in Price $t(s_t)$
0	$2,000	--------
1	$4,000	$7,000
2	$5,000	$6,000
3	$9,000	$2,000
4	$12,000	$1,000
5	--------	$0

$$v_t(s_t) = \min_{a_t \in \{B,K\}} \{p - r(s_t) + c(0) + v_{t+1}(1), c(s_t) + v_{t+1}(s_t + 1)\}, \quad \forall s_t$$

$$v_T(s_T) = -\mathrm{r}(\mathrm{s}_t), \quad \forall s_T$$

ERP as a Shortest Path Problem

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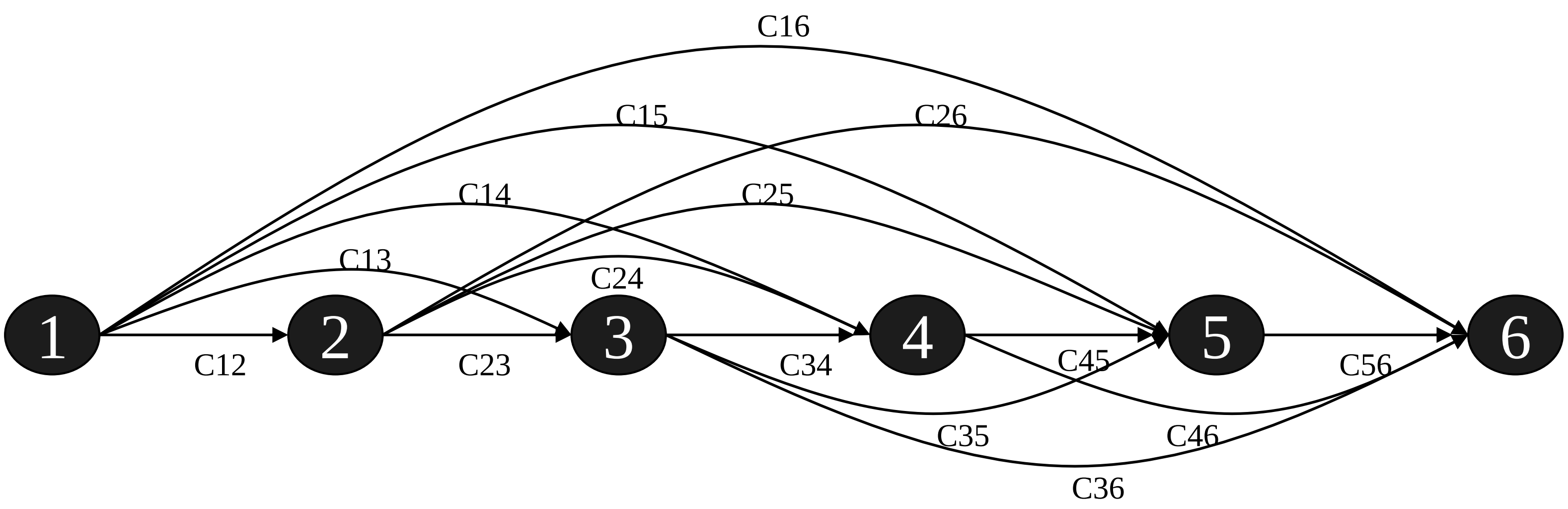
Graph representation:



Optimality Equations:

$$v_i = \min_{j=i+1,\ldots,n} \{c_{ij} + v_j\}, \qquad v_n = 0$$

ERP as a Shortest Path Problem

c_{ij} is the total cost associated with period i given new purchase in period i and j

c_{ij} = operating cost during years $i, i+1, \ldots, j-1$ + purchase cost at the beginning of year i - trade-in value at the beginning of year j

$c_{12}=2+12-7=7$
$c_{13}=2+4+12-6=12$
$c_{14}=2+4+5+12-2=21$
$c_{15}=2+4+5+9+12-1=31$
$c_{16}=2+4+5+9+12+12-0=44$

$c_{23}=2+12-7=7$
$c_{24}=2+4+12-6=12$
$c_{25}=2+4+5+12-2=21$
$c_{26}=2+4+5+9+12-1=31$

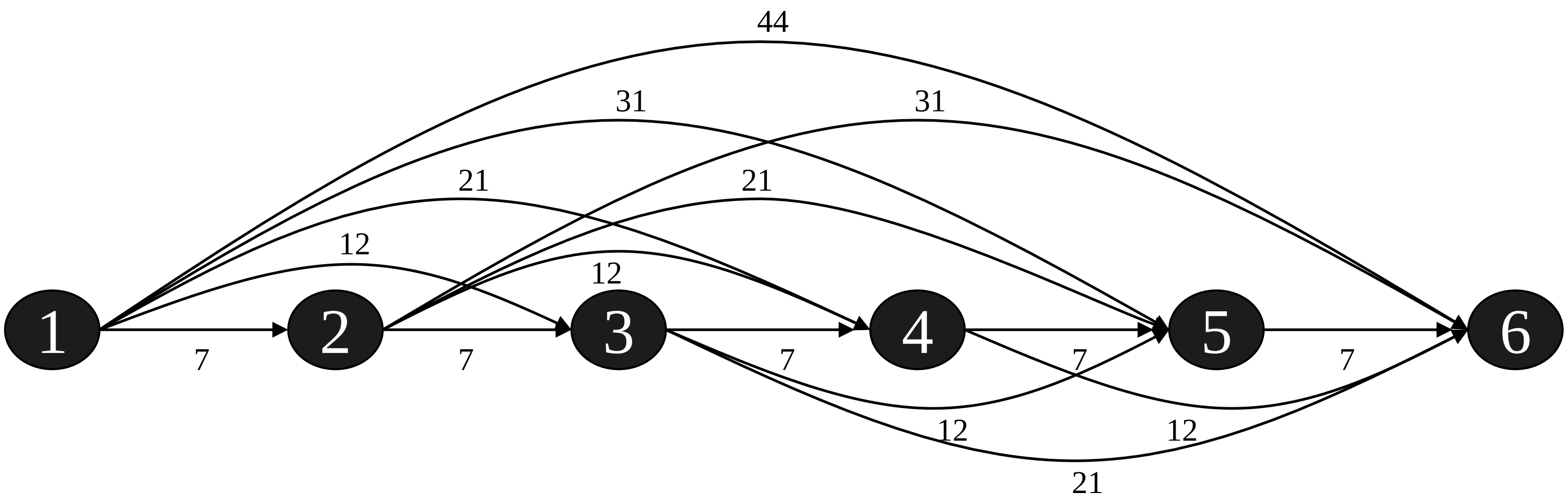
$c_{34}=2+12-7=7$
$c_{35}=2+4+12-6=12$
$c_{36}=2+4+5+12-2=21$

$c_{45}=2+12-7=7$
$c_{46}=2+4+12-6=12$

$c_{56}=2+12-7=7$

Example

Find the shortest path from vertex 1 to 6



Optimality Equations:

$$v_i = \min_{j=i+1,\ldots,n} \{c_{ij} + v_j\}, \qquad v_6 = 0$$

Reading Assgnments

Chapters 1-3 of Dreyfus and Law

Read Lecuture3.computational-effort-example.pdf on canvas for detailed analysis and comparison of the two different forms of the ERP we discussed today.