ISE789 - Stochastic Programming

Assignment # 4 - Due November 23, 2010

Note: When answering assignment questions be sure to show all of your work. Points are allocated for each step of the process.

Question 1 (5 Points): Consider a second stage LP with the following constraints:

$$\begin{aligned}
&y_1 + 3y_2 - y_3 = -6\xi + 5x_1 - x_2 \\
&2y_1 - 1y_2 + 2y_3 + y_4 = -4\xi + 2x_2 + 4x_3 \\
&y_1, y_2, y_3, y_4 \geq 0
\end{aligned}$$

where ξ is a discrete random variable and $\xi \in \Xi = \{0, 1\}$.

Part (a): Write down the linear programs (primal and dual) needed to determine if there is a feasible solution for a given first stage decision x.

Part (b): Describe how these linear programs allow you to obtain an inequality (infeasibility cut) that separates x from $pos(W)$

Part (c): Let $x_1 = x_2 = x_3 = 1$. Find the infeasibility cut.

Question 2 (7 Points): Consider the following stochastic program:

$$\begin{aligned}
Q(x_1, x_2) = \quad & \min x_1 + x_2 + 5 \int_{\xi_1=1}^{4} \int_{\xi_2=1/3}^{1} (y_1(\xi_1, \xi_2) + y_2(\xi_2, \xi_2) d\xi_1 d\xi_2 \\
& \text{s.t. } \xi_1 x_1 + x_2 + y_1(\xi_1, \xi_2) \geq 7, \forall \xi_1, \xi_2 \in \Xi \\
& \xi_2 x_1 + x_2 + y_2(\xi_1, \xi_2) \geq 4, \forall \xi_1, \xi_2 \in \Xi \\
& x_1, x_2 \geq 0 \\
& y_1(\xi_1, \xi_2), y_2(\xi_1, \xi_2) \geq 0
\end{aligned}$$

where $\xi_1 \sim U(1, 4)$, $\xi_2 \sim U(1/3, 1)$, $\Xi = [1, 4] \times [1/3, 1]$

Part (a): Compute the Jensen lower bound for $Q(1, 3)$.

Part (b): Compute the Edmundson-Madansky upper bound for $Q(1.5, 2.5)$.

Part (c): Refine the bounds of part (a) and (b) using the following partition of $\Xi = \{\Xi^1, \Xi^2, \Xi^3, \Xi^4\}$:

$$\Xi^1 = \{\xi_1 \times \xi_2 | 1 \leq \xi_1 \leq 5/2, 1/3 \leq \xi_2 \leq 2/3\}$$

$$\Xi^2 = \{\xi_1 \times \xi_2 | 5/2 \leq \xi_1 \leq 4, 1/3 \leq \xi_2 \leq 2/3\}$$

$$\Xi^3 = \{\xi_1 \times \xi_2 | 1 \leq \xi_1 \leq 5/2, 2/3 \leq \xi_2 \leq 1\}$$

$$\Xi^4 = \{\xi_1 \times \xi_2 | 5/2 \leq \xi_1 \leq 4, 2/3 \leq \xi_2 \leq 1\}$$

Question 3 (6 Points): Consider the following stochastic program:

$$\begin{aligned}
&z^* = \min 8x_1 + 6x_2 + 12x_3 + 7x_4 + E_\xi[35y_{11} + 41y_{21} + \\
&30y_{31} + 5y_{41} + 30y_{12} + 25y_{22} + +19y_{32} + 33y_{42} + 4y_{13} + 5y_{23} + 3y_{33} + 6y_{43}] \\
&\text{s.t. } \textstyle\sum_{i=1}^4 y_{i1} = \xi_1 \\
&\textstyle\sum_{i=1}^4 y_{i2} = \xi_2 \\
&\textstyle\sum_{i=1}^4 y_{i3} = \xi_3 \\
&\textstyle\sum_{i=1}^4 x_i \geq 12 \\
&\textstyle\sum_{j=1}^3 y_{1j} \leq x_1 \\
&\textstyle\sum_{j=1}^3 y_{2j} \leq x_2 \\
&\textstyle\sum_{j=1}^3 y_{3j} \leq x_3 \\
&\textstyle\sum_{j=1}^3 y_{4j} \leq x_4 \\
&10x_1 + 7x_2 + 16x_3 + 6x_4 \leq 120 \\
&x, y \geq 0
\end{aligned}$$

where $\xi_1 \sim U(3,7)$, $\xi_2 \sim U(2,3)$, $\xi_3 \sim U(1,2)$

Part (a): Compute a statistical lower bound, L, on z^*.

Part (b): Compute and provide justification for a 95% confidence interval around the value of L computed in part (a).

Part(c): Compute a statistical upper bound, U, on z^*.

Part (d): Compute and provide justification for a 95% confidence interval around the value of U computed in part (c).

Question 4 (6 Points): Use the Integer L-shaped method to solve the following problem. Show all of your work including the branch and bound tree, cuts, bounds, branching etc. (provide carefully commented code).

$$\begin{aligned}
&z^* = \max 10x_1 + 7x_2 + 11x_3 + 6x_4 - E_\xi[2.1y] \\
&\text{s.t. } \xi x_1 + 4x_2 + 5x_3 + 3x_4 - y \leq 12 \\
&x \in \{0,1\}, y \geq 0
\end{aligned}$$

where

$$\xi = \begin{cases} 1, & \text{w.p } 1/3 \\ 7, & \text{w.p. } 2/3 \end{cases}$$

Question 5 (5 Points): Provide a detailed proof of Proposition 1 in: Laporte et al, 1994 "A Priori Optimization of the Probabilistic Traveling Salesman Problem", *Operations Research*, 42(3), 543-549.

Question 6 (5 Points): Describe in words the "Dilemma" discussed in: Blau, R.A., 1974 "Stochastic Programming and Decision Analysis: An Apparent Dilemma", *Management Science*, 21(3), 271-276. Provide a reasonable enhancement to the model that would 'fix" the problem and show that it results in $EVPI > 0$.

Question 7 (5 Points): From Birge, J., 1985, "Decomposition and Partitioning Methods for Multistage Stochastic Linear Programs", *Operations Research*, 33(5), 989 - 1007:

Part (a): Explain why degeneracy is a problem (see Remark 4, p. 996).

Part (b): Prove proposition 3.