Stochastic Programming
Lecture 7

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September 21, 2010

Today's Class

1. Research Paper Presentations
2. Expected Value of Perfect Information (EVPI)
3. Value of the Stochastic Solution (VSS)
4. Basic Inequalities and EVPI vs. VSS
5. Relationship between VSS and EVPI
6. Bounds on EVPI and VSS

Outline

Research Paper Presentations

Starting in mid October we will begin reviewing research papers in the following tentative areas:

- Statistical sampling
- Multi-Stage Stochastic Linear Programming
- Bounding methods
- Integer L-shaped Method
- Probabilistic Constraints
- Robust Optimization

You can work in groups of 1 or 2. Please prioritize topics you are interested in and I will assign papers to cover.

Outline

Expected Value of Perfect Information (EVPI)

Expected Value of Perfect Information (EVPI) is a measure of how much value could be gained from predicting future outcomes.

Definition:

$$\min z(x,\xi) = \min_x\{cx + \min_y\{q(\omega)^T y | Wy = h(\omega) - T(\omega)x, y \geq 0\} | Ax = b, x \geq 0\}$$

Assuming there is an optimal solution, $\bar{x}(\xi)$, for any $\xi \in \Xi$ define two new problems

- *Wait and See* Solution (WS): $WS = E_\xi[\min z(x,\xi)] = E_\xi[z(\bar{x}(\xi),\xi)]$
- *2-Stage Recourse Problem* (RP): $RP = \min_x E_\xi z(x,\xi)$

Expected Value of Perfect Information (EVPI)

The definition of EVPI is:

$EVPI = RP - WS$

Question: What is the EVPI for the newsvendor problem?

Outline

Value of the Stochastic Solution (VSS)

The *Value of the Stochastic Solution* (VSS) is the difference between the recourse problem (RP) and an "easy alternative":

$$EV = \min_x \{z(x, E[\xi])\}$$

EV is the*mean value problem* (MV), with optimal solution $\bar{x}$.

$$EEV = E_\xi[z(\bar{x}, \xi)]$$

(EEV) is the expected value of using $\bar{x}$ in the RP.

$$VSS = EEV - RP$$

Value of the Stochastic Solution (VSS)

The *Value of the Stochastic Solution* (VSS) is:

$$VSS = EEV - RP$$

What is the VSS for the newsvendor problem?

Outline

Basic Inequalities and EVPI vs. VSS

Some useful propositions from Birge and Louveaux.....

Proposition 1 (Ch. 4 B & L):

$$WS \leq RP \leq EEV$$

Proposition 2 (Ch. 4 B & L):

For stochastic programs with fixed objective coefficients, fixed T, and fixed W,

$$EV \leq WS$$

Basic Inequalities and EVPI vs. VSS

Proposition 3 (Ch. 4 B & L):

Let x^* represent an optimal solution to the RP problem and let $\bar{x}(\bar{\xi})$ be an optimal solution to the EV problem then

$$RP \geq EEV + (x^* - \bar{x}(\xi))\eta$$

where $\eta \in \partial E_\xi[z(\bar{x}(\bar{\xi}, \xi)]$, the subdifferential set of $E_\xi[z(\bar{x}(\bar{\xi}, \xi)]$ at $\bar{x}(\bar{\xi})$.

Basic Inequalities and EVPI vs. VSS

Consider the following (slighted revised version of the recourse) problem:

Proposition 4 (Ch. 4 B & L):

$$\min z_u(x,\xi) = cx + \min\{qy \mid Wy \geq h(\xi) - Tx, y \geq 0\} \tag{1}$$

$$s.t. Ax = b \tag{2}$$

$$x \geq 0 \tag{3}$$

and the following definition of the RP

$$RP = \min_x \{E_\xi[z_u(x,\xi)]\}$$

Assume further that $h(\omega)$ is bounded above by h_{max}. Let x_{max} be an optimal solution to $z_u(x, h_{max})$. Then

$$RP \leq z_u(x_{max}, h_{max}).$$

Outline

Relationship between VSS and EVPI

EVPI and VSS measure different things and can have very different values. The following can be proved:

Proposition 5 (Ch. 4 B & L):

For any stochastic program:

$$EVPI \geq 0, \quad VSS \geq 0$$

and for stochastic programs with fixed recourse and fixed objective function coefficients

$$EVPI \leq EEV - EV, \quad VSS \leq EEV - EV$$

Outline

Bounds on EVPI and VSS

Consider a simplified version of a stochastic program where $\xi = h(\omega)$) and Ξ is finite with scenarios $\xi^k, k = 1, ..., K$. To develop the bounds we consider a *reference scenario*, ξ^r. Common reference scenarios are:

- expected value: $\bar{\xi}$
- *worst case scenario*

Let $p^r = P(\xi = \xi^r)$ and define the following to be the *PAIRS Subproblem*:

$$\min z^P(x, \xi^r, \xi^k) = cx + p^r qy(\xi^r) + (1 - p^r)qy(\xi^k) \tag{4}$$

$$s.t. \quad Ax = b \tag{5}$$

$$Wy(\xi^r) = \xi^r - Tx \tag{6}$$

$$Wy(\xi^k) = \xi^k - Tx \tag{7}$$

$$x, y \geq 0. \tag{8}$$

Bounds on EVPI and VSS

Proposition 6 (Ch. 4 B & L):

When the reference scenario is not in Ξ, then $SPEV = WS$, where $SPEV$ is the sum of pairs expected values problem:

$$SPEV = \frac{1}{1-p^r}\sum_{k=1}^{K} p^k \min z^P(x, \xi^r, \xi^k)$$

Bounds on EVPI and VSS

Proposition 7 (Ch. 4 B & L):

$$WS \leq SPEV \leq RP$$

Proposition 8 (Ch. 4 B & L):

$$RP \leq EPEV \leq EVRS$$

Proposition 9 (Ch. 4 B & L):

$$0 \leq EVRS - SPEV \leq VSS \leq EVRS - SPEV \leq EVRS - WS$$