ISE789: Stochastic Programming
Lecture 1

Dr. Brian Denton

NCSU

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Today's Class

1. Introductions
2. Course Details
3. Stochastic Programming Background
4. The Newsvendor Model

Outline

About me....

- PhD in Operations Research
- 4 years R & D at IBM
- 2 years at Mayo Clinic
- 3 years at NC State
- Research Interests: Optimization under uncertainty; health care delivery; chronic disease treatment decisions

Outline

Course Details

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Office Hours: By appointment
Teaching Assistant: Also me

Course Goals:
For you to learn about stochastic programming (SP) and ....

- Understand the theory underlying SP
- Be able to implement state-of-the art algorithms
- Be able to integrate SP into your research

Textbook

We will use “Introduction to Stochastic Programming”, by Birge and Louveaux. Following are other references that you may find useful:

- “Stochastic Programming,” Kall and Wallace, 1995 (available online, see the syllabus)
- “Lectures on Stochastic Programming, ” Shapiro and Ruszczynski, 2003 (available online, see the syllabus)
- “Stochastic Programming, Handbooks in OR,” Ruszczynski and Shapiro (Eds.), 2003

We’ll also go through many journal articles (after all, this is a research course)

See the course web site for additional supplemental reading

Grades

The following will be used to compute your final grade:

- Assignments (Total: 40%)
- A research project (50%)
- Class participation (10%): This includes review of one or more papers

Note: I reserve the right to hold a final exam worth 30% of the final grade

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The most important part (to me) is the research project. The project will involve:

- A proposal: due Thursday, September 30th: 20%
- A final report including: Introduction, Literature Review, Evaluation of a proposed solution method, Numerical results, Conclusions: 70%
- In class presentation of your Proposal and Project: 10%

Course Expectations

You:

- Assignments, final exam, and projects are not collaborative, what you hand in should be your own work
- Hand in your work on time
- Show up for class on time and learn something (or look like you are)
- Participate in class

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Me:

- Show up for class
- Teach
- Give you timely feedback about your grades

Outline

Stochastic Programming Background

Stochastic programming (SP) involves decision making under uncertainty. (What does “stochastic” mean? What does “programming” mean?)

Why study stochastic programs? Isn’t math programming hard enough already?

Some history:

- Almost as old as linear programming. The first papers appeared in the 1950’s (Dantzig, G. 1955, “Linear Programming Under Uncertainty”, Management Science)
- Related to other methods for decision making under uncertainty including: decision analysis, discrete time optimal control, robust optimization

Stochastic Programming Background

Most SPs have an element of time in the decision process. The simplest case is the *2-stage recourse problem*:

- One or more first stage decisions, x
- One or more random variables, ξ
- A second stage recourse (response) decision $y(\xi)$

We'll start by looking at the simplest 2-stage problem....

Outline

Model Formulation

Description: A state health department must decide how many doses of flu vaccine to purchase for the upcoming flu season. The per unit cost is c, and reimbursement for a flu shot is p. Unsold doses can be returned with salvage value r where $r < c$. Let ξ denote random annual demand for flu shots with probability density function (pdf), $f(\xi)$, and cdf $F(\cdot)$.

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Classic formulation:
Let $Q(x, \xi)$ denote the cost for a particular choice of x and realization of ξ:

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Optimization Problem: $\min_x \{Q(x) = E_\xi[Q(x, \xi)]\}$

$Q(x)$ can be shown to be continuous and convex and $x^* = F^{-1}(\frac{p-c}{p-r})$.

Model Formulation

An Alternative Formulation:

Using the fact that $x = \xi + (x - \xi)^+ - (\xi - x)^+$ it follows that:

$$\begin{aligned} Q(x,\xi) &= c\xi + c(x-\xi)^+ - c(\xi - x)^+ - p\xi + p(\xi - x)^+ - r(x-\xi)^+ \\ Q(x) &= E_\xi[Q(x,\xi)] = (c-p)\mu + E_\xi[(c-r)(x-\xi)^+ + (p-c)(\xi-x)^+] \end{aligned}$$

Letting $c^u = p - c$ and $c^o = c - r$ gives:

$$x^* = \arg\min_x \{E_\xi[c^u(\xi - x)^+ + c^o(x-\xi)^+]\}$$

Model Formulation

Using Leibniz's rule (remember this?)....

$$\frac{\partial}{\partial x}\int_{a(x)}^{b(x)} f(\xi,x)d\xi = \int_{a(x)}^{b(x)} \frac{\partial f(\xi,x)}{\partial x}d\xi + f(b(x),x)\frac{\partial b}{\partial x} - f(a(x),x)\frac{\partial a}{\partial x}$$

the first term is:

$$c^u\frac{\partial}{\partial x}\int_{x}^{\infty}(\xi - x)f(\xi)d\xi = c^u\int_{x}^{\infty}(-1)f(\xi)d\xi = -c^u(1-F(x))$$

and the second term is:

$$c^o\frac{\partial}{\partial x}\int_{0}^{x}(x-\xi)f(\xi)d\xi = c^o\int_{0}^{x}f(\xi)d\xi = c^oF(x)$$

reorganizing these we find that $x^* = F^{-1}(\frac{c^u}{c^u+c^o})$

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Pros: It is much easier to solve: $x^* = E_\xi[\xi]$

Cons: It may be suboptimal. Why?

Wait-and-See Solution

The **wait-and-see problem** (WS) assumes you have perfect information about the future

$$WS = E_{\xi}[\min_{x(\xi)}\{c^u(\xi - x(\xi))^+ + c^o(x(\xi) - \xi)^+\}]$$

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Unfortunately most SPs don't have nice closed form solutions like the newsvendor problem.

Properties of the Newsvendor Model

The following are important properties that we skipped over in our analysis of the newsvendor problem (we'll talk about them in a more general context later)....

1) Convexity: The newsvendor problem has an objective function that is the expectation of $Q(x, \xi) = \min_x \{c^u(\xi - x)^+ + c^o(x - \xi)^+\}$. For a particular ξ this is easily proved to be (piecewise linear) convex. Convexity of $Q(x)$ follows in expectation (under weak assumptions).

2) Continuity: $Q(x, \xi)$ is continuous. Continuity of $Q(x)$ follows (under weak assumptions).

3) Differentiability: $Q(x, \xi)$ is not differentiable everywhere (but sometimes $Q(x)$ is!)

Properties of the Newsvendor Model

4) Feasibility: $Q(x, \xi)$ is feasible for any x.

5) Optimal Solution: There is a closed form expression for the optimal solution to $\min_x\{Q(x)\}$.

Under weak assumptions (1) and 2), are generally true for most stochastic linear programs, (3) is true if the random variables have continuous support, (4) is sometimes true, and (5) is almost never true.

Homework: Survey

- **September 7 class is cancelled**
- Start reading chapter 1 of B & L
- Complete and hand in your surveys before next class
- Use the last question on the survey to express any particular interests about material you would like to cover in this class