Stochastic Programming Models for Optimization of Surgery Delivery Systems

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Summary

Introduction

Surgery process and complicating factors

Examples:
- Problem 1: Single OR scheduling
- Problem 2: Multi-OR surgery allocation
- Problem 3: Bi-criteria scheduling of a surgery suite

OR = Operating Room

Other Related Research
Motivation

- Health care expenditures in the United States exceeded $2 trillion in 2012
- Surgery accounts for the single largest proportion of a U.S. hospital’s total expenses and revenues
- Efficient access to surgery is important for patient health and safety
Surgery in the U.S. is performed in two types of facilities:

- **Hospitals**
  - Open 24 hours a day
  - Patient recover in the hospital
  - Complex surgeries

- **Ambulatory Surgery Centers**
  - Normally open 7am to 5pm
  - Patients admitted and discharged same day
  - Lower cost and lower infection rate than hospitals
Surgery Process

**Patient Intake:** administrative activities, pre-surgery exam, gowning, site prep, anesthetic

**Surgery:** incision, one or multiple procedures, pathology, closing

**Recovery:** post anesthesia care unit (PACU), ICU, hospital bed
The work I will discuss is motivated by problems at Mayo Clinic in Rochester, MN, U.S.:
Ambulatory Surgery Center

A typical ambulatory surgery center:

Patient waiting area

Operating rooms

Recovery rooms

Blue lines represent patient flow
Decisions that can be supported with operations research models:

- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling of staff and patients in intake, surgery, and recovery
- How to design the suite (intake rooms, recovery rooms, ORs)
Complicating Factors

- Many types of resources to be scheduled: surgery team, equipment, materials
- **High cost of resources** and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- **Uncertainty** in duration of activities
- Many **competing criteria**
Time for surgery is random

Empirical distribution for tonsilectomy:

Minutes

0 0.05 0.1 0.15 0.2 0.25
10 25 40 55 70 85 100
Empirical distribution for abdominal surgery to repair a hernia:
Problem 1: Single Operating Room Scheduling
Problem Description:

Given a single operating room with multiple surgeries to be completed, what is the optimal amount of time to allocate for each surgery to minimize the cost of:

- Waiting of patients and surgery teams to start surgery
- Unutilized (idle) time of the operating room
- Overtime with respect to a fixed length of day
Single OR Scheduling

Planned OR Time (e.g. 8 hours)

Example Scenario:

Goal: Min\{ Idling + Waiting + Overtime\}
Problem: Find the planned time for surgeries to minimize the objective

\[
\min_{x} \left\{ \sum_{i=1}^{n} C_i^w E_Z[W_i] + \sum_{i=1}^{n} C_i^s E_Z[S_i] + C_i^L E_Z[L] \right\}
\]

\[
W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)
\]

\[
S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)
\]

\[
L = \max(W_n + Z_n + \sum x_i - d, 0)
\]

Finding the expectation of waiting, idling, and overtime is difficult.
Literature Review – Single Server

Queuing Analysis:
- Mercer (1960, 1973)
- Jansson (1966)
- Brahimi and Worthington (1991)

Assumes steady state is reached, and arrivals are random

Heuristics:
- White and Pike (1964)
- Soriano (1966)
- Ho and Lau (1992)

Does not guarantee optimal solution

Optimization:
- Weiss (1990) – 2 surgery news vendor model
- Denton and Gupta (2003) – General stochastic programming formulation
The problem can be reformulated as a two stage stochastic linear program:

\[
\min \left\{ E_Z \left[ \sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c_i^s s_i + c^L l \right] \right\}
\]

s.t. \[ w_2 - s_2 = Z_1 - x_1 \]
\[ -w_2 + w_3 - s_3 = Z_2 - x_2 \]
\[ -w_n - s_n + l - g = Z_n - d + \sum_{j=1}^{n-1} x_i \]

\[ x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1, \ldots, n, \quad l, g \geq 0 \]
Two Stage Recourse Problem

Initial Decision (x) $\rightarrow$ Uncertainty Resolved $\rightarrow$ Recourse (y)

$$\min \{ Q(x) = E_z[Q(x, Z)] \}$$

$$Q(x, Z^k) = \min \{ c \cdot y^k \mid T \cdot x + W \cdot y^k = h^k, y^k \geq 0 \}$$

Solve using outer linearization
Example

Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with uniform distribution, U(1,2):
General Insights

We can draw the following conclusions:

- Simple heuristics often perform poorly
- The value of the stochastic solution (VSS) is high
- Large instances of this problem can be solved very quickly using decomposition


Problem 2: Multiple Operating Room Surgery Allocation
**Problem Description:**

Given a set of surgeries to be scheduled on a certain day decide the following:

- How many operating rooms to make available to complete all surgeries
- Which operating room to perform each surgery block (set of surgeries completed by a given surgeon)
Multi-Operating Room Scheduling

Decisions:

- How many operating rooms (ORs) to open each day?
- Which OR to schedule each surgery block in?

Performance Measures:

- Cost of operating rooms opened
- Overtime costs for operating rooms
Extensible Bin-Packing

\[ x_i = \begin{cases} 
1 & \text{if OR } i \text{ active} \\
0 & \text{otherwise}
\end{cases} \quad y_{ij} = \begin{cases} 
1 & \text{if surgery } j \text{ assigned to OR } i \\
0 & \text{Otherwise}
\end{cases} \]

\[ Z = \min \{ \sum_{i=1}^{m} c^f x_i + c^v o_i \} \]

s.t. \[ y_{ij} \leq x_i \quad i = 1, \ldots, m, \quad j = 1, \ldots, n \]

\[ \sum_{i=1}^{m} y_{ij} = 1 \quad j = 1, \ldots, n \]

\[ \sum_{j=1}^{n} p_j y_{ij} - o_i \leq dx_i \quad i = 1, \ldots, m \]

\[ y_{ij}, x_i \text{ binary}, \quad o_i \geq 0 \]

Cost of ORs + Overtime

Surgeries only scheduled in ORs that are active

Every surgery goes in one OR

Overtime if surgery goes past end of day, T
Symmetry

There are m! optimal solutions:

OR Ordering

Adding the following anti-symmetry constraints reduces computation time:

\[
x_1 \geq x_2 \\
x_2 \geq x_3 \\
\vdots \\
x_m \geq x_{m-1}
\]

\[
y_{11} = 1 \\
y_{21} + y_{22} = 1 \\
\vdots \\
\sum_{j=1}^{m} y_{mj} = 1
\]

Surgery Assignment
The real problem is stochastic due to random surgery durations.

\[ Q(\mathbf{x}) = \min \left\{ \sum_{j=1}^{m} c^f x_j + c^v E_\omega [o_j(\omega)] \right\} \]

s.t. \( y_{ij} \leq x_j \quad \forall (i, j) \)

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} Z_i(\omega) y_{ij} - o_j(\omega) \leq dx_j \quad \forall (i, j, \omega) \]

\( y_{ij}, x_j \in \{0, 1\}, \quad o_j(\omega) \geq 0, \forall \omega \)
Integer L-Shaped Method

Branch and bound tree:

This problem can be solved using decomposition

Master Problem:

\[ Z = \min \{ \sum_{j=1}^{m} c^f x_j + \Theta \} \]

s.t. \( y_{ij} \leq x_j \quad \forall (i, j) \)

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\( y_{ij}, x_j \in \{0,1\}, \Theta \geq 0 \)

(optimality cuts)

\[ \Theta \geq E_\omega [\pi (h - Tx)] \]
Heuristic and Bounds

Dell’Ollmo (1998) – provides a 13/12 approximation algorithm for bin packing with extensible bins

Heuristic:

Start with lower bound (LB)

\[ n \leftarrow LB; \]
\[ \text{repeat}; \]
\[ LPT(n); \]
\[ \text{if } (o_j = 0, \forall j) \text{ Stop}; \]
\[ n \leftarrow n+1; \]
\[ \text{end(repeat);} \]

\[
LB = \left[ \frac{\sum_{i=1}^{n} z_i}{d(1 + \frac{c_f}{c^v d})} \right]
\]

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room
Robust Formulation

Robust formulation seeks to minimize the worst case cost.

\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + F(x, y) \right\} \]
\[ s.t. \quad y_{ij} \leq x_j \quad \forall (i, j) \]
\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]
\[ y_{ij}, x_j \in \{0,1\} \geq 0 \]

Worst case (adversary) problem

\[ F(x, y) = \begin{cases} 
\max_{\delta} \left\{ \sum_{j=1}^{m} \eta_j \right\} \\
\quad \text{s.t.} \quad \eta_j = c^r \max \{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - dx_j\}, \quad \forall j \\
\quad \sum_{(i,j):y_{ij}=1} \frac{\delta_{ij} - z_i}{z_i - \underline{z}_i} y_{ij} \leq \tau \\
\quad \underline{z}_i \leq \delta_{ij} \leq z_i, \quad \forall (i, j) : y_{ij} = 1 
\end{cases} \]

Uncertainty budget controls how conservative the solution is.
Results for sample test problems based on Mayo Clinic surgery center:

<table>
<thead>
<tr>
<th>Instance</th>
<th>MV_IP</th>
<th>LPT_Heu</th>
<th>Tau=2</th>
<th>Tau=4</th>
<th>Tau=6</th>
<th>MV_IP</th>
<th>LPT_Heu</th>
<th>Tau=2</th>
<th>Tau=4</th>
<th>Tau=6</th>
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<tr>
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<td>0.806</td>
<td>0.892</td>
<td>0.906</td>
<td>0.933</td>
<td>0.999</td>
<td>0.998</td>
<td>0.880</td>
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<td>2</td>
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<td>0.966</td>
<td>0.898</td>
<td>0.896</td>
<td>0.970</td>
<td>0.999</td>
<td>0.999</td>
<td>0.999</td>
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<td>0.852</td>
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<tr>
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<td>0.998</td>
<td>0.930</td>
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<tr>
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<td>0.910</td>
<td>0.893</td>
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<td>0.996</td>
<td>0.900</td>
<td>0.901</td>
<td>0.903</td>
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<td>0.922</td>
<td>0.988</td>
<td>0.993</td>
<td>0.916</td>
<td>0.951</td>
<td>0.933</td>
</tr>
<tr>
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<td>0.047</td>
<td>0.028</td>
<td>0.046</td>
<td>0.013</td>
<td>0.010</td>
<td>0.059</td>
<td>0.045</td>
<td>0.028</td>
</tr>
</tbody>
</table>

MV_IP = solution of mean value problem, LPT_Heu = application of longest processing time first heuristic. Results expressed as the ratio of optimal solution to solution generated by MV_IP, LPT_Heu, Robust IP.
General Insights

A fast LPT based heuristic works well on a large number of instances

- LPT works well when overtime costs are low
- LPT is better (and easier) than solving MV problem in most cases

Robust IP is better than LPT when overtime costs are high

Problem 3: Patient Arrival Scheduling
Problem Description:

Find the efficient frontier of appointment times for patients having a procedure in an ambulatory surgery center to trade off:

- Expected patient waiting time prior to surgery
- Expected length of day to complete all surgeries
Endoscopy suite provides minimally invasive procedures to screen for cancer:
Probability distributions for intake (preparation), surgery, and recovery from surgery, based on 1 year of data from Mayo Clinic:

Mean time for surgery about 20 minutes

Mean time for recovery about 45 minutes
Decision variables: scheduled start times to be assigned to $n$ patients each day

Goal: Generate the efficient frontier of schedules to understand tradeoffs between patient waiting and length of day

- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA
The non-dominated sorting genetic algorithm (NSGA-II) of Deb et al. (2000) is used in the simulation optimization.
Selection Procedure

Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) is used to compute the number of samples necessary to determine whether a solution \( i \) “dominates” \( j \)

Solution \( i \) “dominates” \( j \) if:

\[
E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]
\]
Main features of the GA:

- Randomly generated initial population of schedules
- Selection based on 1) ranks and 2) crowding distance
- Single point crossover:

Parents

\[
\begin{array}{cccc}
  z_1 & z_2 & z_3 & \ldots & z_n \\
  y_1 & y_2 & y_3 & \ldots & y_n \\
\end{array}
\]

Children

\[
\begin{array}{cccc}
  z_1 & z_2 & - & y_3 & \ldots & y_n \\
  y_1 & y_2 & - & z_3 & \ldots & z_n \\
\end{array}
\]

- Mutation
Example of the progression of the genetic algorithm
General Insights

We drew the following conclusions from our study:

- The simulation optimization approach provides significant improvement to schedules used in practice.
- Controlling the mix of different types of surgery each day can significantly improve both patient waiting time and overtime.

Other Research

There are many opportunities for future research. Following are three brief examples of other related research:

- Scheduling when Operating Rooms are treated as a shared resource among surgeons
- Dynamic (online) scheduling
- Scheduling of other parts of the hospital
A more advanced model can be used to evaluate the benefit of surgeons sharing operating rooms as a pooled resource.

Dynamic Scheduling

In some healthcare environments patient requests are made dynamically and the number of patients is uncertain:

- Patients request appointments stochastically
- Appointment decisions are made one at a time
- Formulated as a multistage stochastic linear program

Optimization models can be applied to other parts of the hospital such as cancer centers:

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More details about the papers discussed can be found at my website:

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Thank You

ありがとうございました