Optimization of Surgery Delivery Systems

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March 23, 2009
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Supported by National Science Foundation – CMMI 0620573
Summary

- Surgery process and complicating factors
- Stochastic Programming:
  - Problem 1: Single OR scheduling
  - Problem 2: Multi-OR Surgery Allocation
- Simulation Optimization:
  - Problem 3: Bi-criteria scheduling of an Outpatient Procedure Center
- Future research
Surgery Process

- **Patient Intake**: administrative activities, pre-surgery exam, gowning, site prep, anesthetic

- **Surgery**: incision, one or multiple procedures, pathology, closing

- **Recovery**: post anesthesia care unit (PACU), ICU, hospital bed
Outpatient Procedure Center
Outpatient Suite Process Map

**INTAKE**
- Patient arrives at the hospital lobby
- Patient walks down or taken to the operation room
- Patient notifies the lobby front desk that s/he is here
- Patient may wait to go to the operation room

**SURGERY**
- Patient is put on the OR bed and waits for the OR team to arrive
- Patient is given IV if needed and monitored
- Surgeon arrives at the OR. Patient gives consent for operation to the surgeon.
- Patient is sedated
- Patient is intubated
- Patient is extubated

**RECOVERY**
- Patient is discharged
- Patient recovers in the recovery area
- Patient is taken to the recovery area
- Patient may wait to go to the recovery area
Complicating Factors

- Many types of resources to be scheduled: OR team, equipment, materials
- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Many competing criteria
Surgery Duration Uncertainty

Minutes

Hernia
Surgery Duration Uncertainty
Problem 1: Single OR Scheduling
Single OR Scheduling - $S(n)/G(n)/1$

Planned OR Time

$x_1$ $x_2$ $x_3$ $x_4$ $x_5$

$a_1$ $a_2$ $a_3$ $a_4$ $a_5$

Idling

Waiting

Overtime

Min{ Idling + Waiting + Overtime}
Stochastic Optimization Model

\[
\min \left\{ \sum_{i=1}^{n} C_i^w \cdot E[Z[W_i]] + \sum_{i=1}^{n} C_i^s \cdot E[Z[S_i]] + C_L \cdot E[Z[L]] \right\}
\]

\[
W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)
\]

\[
S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)
\]

\[
L = \max(W_n + Z_n + \sum x_i - d, 0)
\]
Literature Review – Single Server

- **Queuing Analysis:**
  - Mercer (1960, 1973)
  - Jansson (1966)
  - Brahimi and Worthington (1991)

- **Heuristics:**
  - White and Pike (1964)
  - Soriano (1966)
  - Ho and Lau (1992)

- **Optimization:**
  - Weiss (1990) – 2 surgery news vendor model
Stochastic Linear Program

\[
\min \{ E_Z \left[ \sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c_s^s s_i + c^L l \right] \}
\]

s.t.

\[
\begin{align*}
  w_2 & \quad - s_2 \\
  - w_2 + w_3 & \quad - s_3 \\
  - w_n & \quad - s_n + l - g = Z_n - d + \sum_{j=1}^{n-1} x_i \\
\end{align*}
\]

\[x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1, \ldots, n, \quad l, g \geq 0\]
Two Stage Recourse Problem

Initial Decision ($x$) $\rightarrow$ Uncertainty Resolved $\rightarrow$ Recourse ($y$)

$$\min\{Q(x) = E_Z[Q(x, Z)]\}$$

$$Q(x) = \sum_{k=1}^{K} p^k Q(x, Z^k)$$

$$Q(x, Z^k) = \min\{c \cdot y^k \mid T x + W y^k = h^k, y^k \geq 0\}$$
Example

• Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with U(1,2):
Surgical Suite Decisions

- Number of cases to schedule
- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling of staff and patients in intake, surgery, and recovery
- How to design the suite (intake rooms, recovery rooms, ORs)
- Selection of equipment resources (surgical kits, diagnostic equipment)
Problem 2: Multi-OR Surgery Allocation
Multi-Operating Room Scheduling

Decisions:
- How many operating rooms (ORs) to open?
- Which OR to schedule each surgery block in?

Performance Measures:
- Cost of operating rooms opened
- Overtime costs for operating rooms
Extensible Bin Packing

\[ x_j = \begin{cases} 
1 & \text{if OR } j \text{ open} \\
0 & \text{if OR } j \text{ closed} 
\end{cases} \]

\[ y_{ij} = \begin{cases} 
1 & \text{if Surg.Block } y \text{ i assigned to OR } j \\
0 & \text{Otherwise} 
\end{cases} \]

\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + c^o o_j \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} z_i y_{ij} - o_j \leq d_j x_j \quad \forall (i, j) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j \geq 0 \]
Symmetry

- $m!$ optimal solutions:

- Anti-symmetry constraints:

\begin{align*}
  x_1 &\geq x_2 \\
  x_2 &\geq x_3 \\
  &\vdots \\
  x_m &\geq x_{m-1}
\end{align*}

\begin{align*}
  y_{11} &= 1 \\
  y_{21} + y_{22} &= 1 \\
  &\vdots \\
  \sum_{j=1}^{m} y_{mj} &= 1
\end{align*}
Two-Stage Stochastic MIP

\[ Q(\mathbf{x}) = \min \{ \sum_{j=1}^{m} c^f x_j + c^p E_\omega [o_j(\omega)] \} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} Z_i(\omega) y_{ij} - o_j(\omega) \leq d_j x_j \quad \forall (i, j, \omega) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \geq 0, \forall \omega \]
Integer L-Shaped Method

Master Problem:

\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + \Theta \right\} \]

s.t. \( y_{ij} \leq x_j \quad \forall (i, j) \)

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\( y_{ij}, x_j \in \{0,1\}, \Theta \geq 0 \)

\( \Theta \geq E_\omega [\pi(h-Tx)] \)
Heuristic and Bounds

Dell’Ollmo (1998) – 13/12 approximation algorithm for bin packing with extensible bins

EBP Heuristic:

\[ n \leftarrow LB; \]
\[ \text{repeat}; \]
\[ \quad LPT(n); \]
\[ \quad \text{if } (o_j = 0, \forall j) \quad \text{Stop}; \]
\[ \quad n \leftarrow n + 1; \]
\[ \text{end(\text{repeat});} \]

\[ LB = \left[ \sum_{i=1}^{n} \bar{z}_i \over T(1 + \frac{c^f}{c^vT}) \right] \]

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room
Robust Formulation

\[ Z = \min \left\{ \sum_{j=1}^{m} c^f x_j + F(x, y) \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ y_{ij}, x_j \in \{0, 1\} \geq 0 \]

\[ F(x, y) = \begin{cases} 
\max_{\delta} \left\{ \sum_{j=1}^{m} \eta_j \right\} \\
\text{s.t.} \quad \eta_j = c^v \max\{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - T_j x_j\}, \text{ } \forall j \\
\sum_{(i,j):y_{ij}=1} \frac{\delta_{ij} - \bar{z}_i}{\bar{z}_i - \bar{z}_i} y_{ij} \leq \tau \\
\bar{z}_i \leq \delta_{ij} \leq \bar{z}_i, \forall (i, j) : y_{ij} = 1 
\end{cases} \]
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<th>MV_IP</th>
<th>LPT_Heu</th>
<th>Tau=2</th>
<th>Tau=4</th>
<th>Tau=6</th>
<th>MV_IP</th>
<th>LPT_Heu</th>
<th>Tau=2</th>
<th>Tau=4</th>
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15 surgery instances
Variable Cost = 0.033 Variable Cost = 0.0083

Robust IP Robust IP

```
General Insights

• The fast LPT based heuristic works (fairly) well on a large number of instances
  – LPT works very well when overtime costs are low
  – LPT is better (and easier) than solving MV problem in most cases
• Robust IP is better than LPT when overtime costs are high
Current Research: Share ORs

Surgeon 1
- 1
- 2
- 3
- 4
- 5

Surgeon 2
- 1
- 2
- 3
- 4

Surgeon 3
- 1
- 2

OR 1

OR 2

Surgeon Turnover Time

Surgeon Idle Time

Overtime

OR Turnover Time

Surgeon Turnover Time
Problem 3: Patient Arrival Scheduling
Endoscopy Suite

Patient Waiting Time

1st Patient Arrival

Length of Day

nth Patient Completion

Preoperative Waiting Area

Operating Rooms

Recovery Area

Patient Check-in Waiting Area

Intake Area

Patient Arrivals

Schedule

Patient Discharge
Intake, Surgery, and Recovery
Simulation-optimization

- **Decision variables**: scheduled start times to be assigned to $n$ patients each day
- **Goal**: Generate the set of non-dominated schedules to understand tradeoffs between waiting and length of day
- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA
Pareto Set

- The non-dominated sorting genetic algorithm (NSGA-II) of Deb et al. (2000) is used in the simulation optimization.
Selection Procedure

• Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) is used to compute the number of samples necessary to determine whether a solution $i$ “dominates” $j$

• Solution $i$ “dominates” $j$ if:

$$E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]$$
Genetic Algorithm

- Main features of the GA:
  - Randomly generated initial population of schedules
  - Selection based on 1) ranks and 2) crowding distance
  - Single point crossover:
    \[
    \begin{align*}
    z_1 & \quad z_2 & \quad z_3 & \ldots & \quad z_n \\
    y_1 & \quad y_2 & \quad y_3 & \ldots & \quad y_n
    \end{align*}
    \]
    \[
    \begin{align*}
    z_1 & \quad z_2 & \quad - & \quad y_3 & \ldots & \quad y_n \\
    y_1 & \quad y_2 & \quad - & \quad z_3 & \ldots & \quad z_n
    \end{align*}
    \]
  - Mutation
Example

Solutions in Criteria Space

[Graph showing the relationship between session length and waiting time with two schedules marked: GA and SA schedule]
Current and Future Research

• Investigating new stochastic programming and robust optimization formulations and methods
• Dynamic (online) scheduling problems
• Surgical suite design and re-configuration
• Real time control on the day of surgery
Questions?