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Optimization of Surgery Delivery Systems

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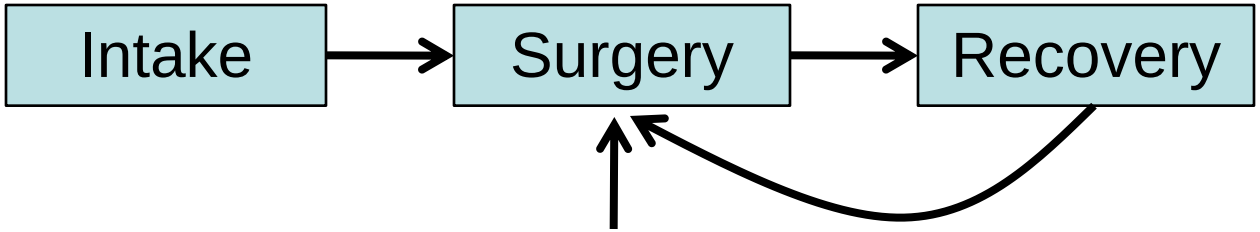
Summary

- Surgery process and complicating factors
- Stochastic Programming:
  - Problem 1: Single OR scheduling
  - Problem 2: Multi-OR Surgery Allocation
- Simulation Optimization:
  - Problem 3: Bi-criteria scheduling of an Outpatient Procedure Center
- Future research

Surgery Process

- Patient Intake: administrative activities, pre-surgery exam, gowning, site prep, anesthetic
- Surgery: incision, one or multiple procedures, pathology, closing
- Recovery: post anesthesia care unit (PACU), ICU, hospital bed



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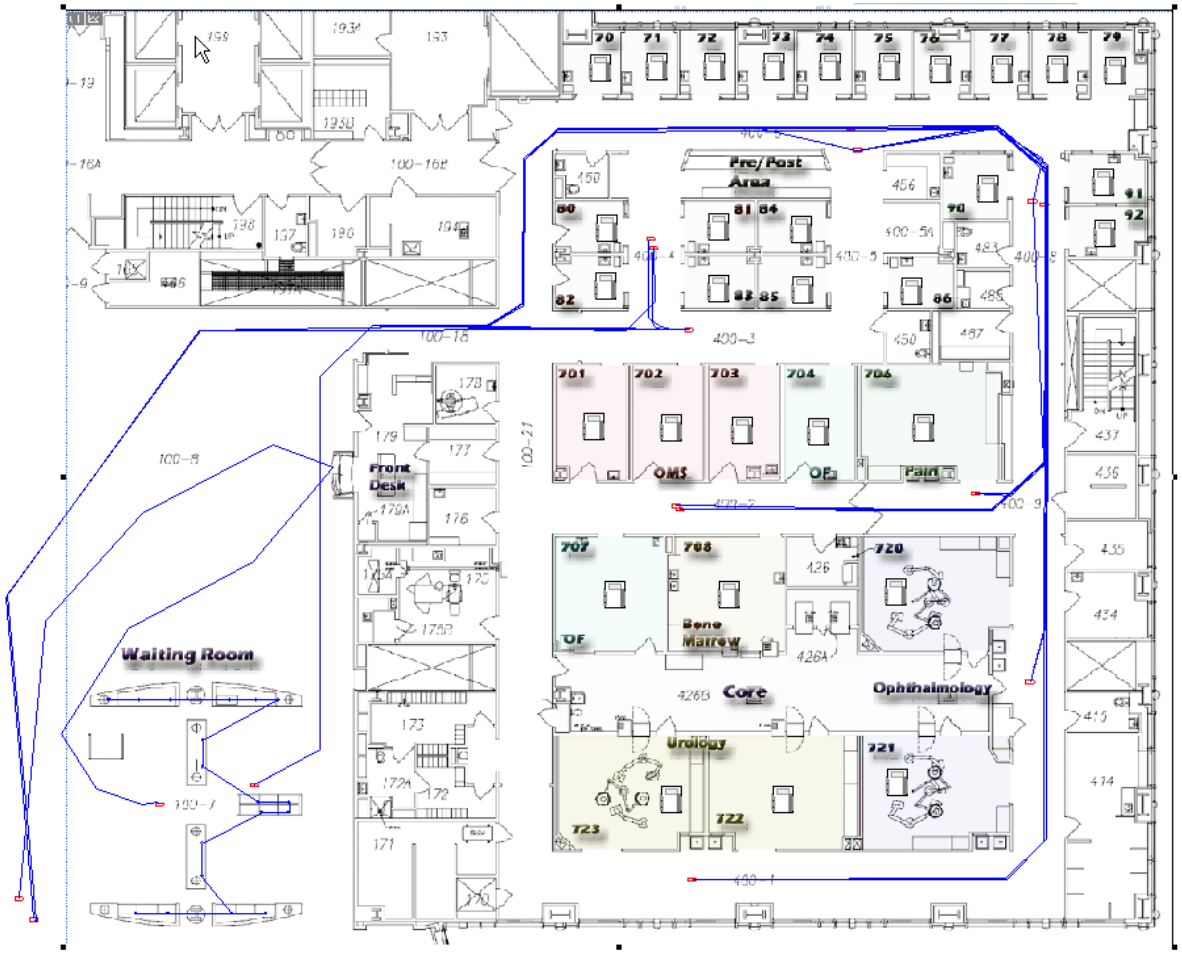
Outpatient Procedure Center



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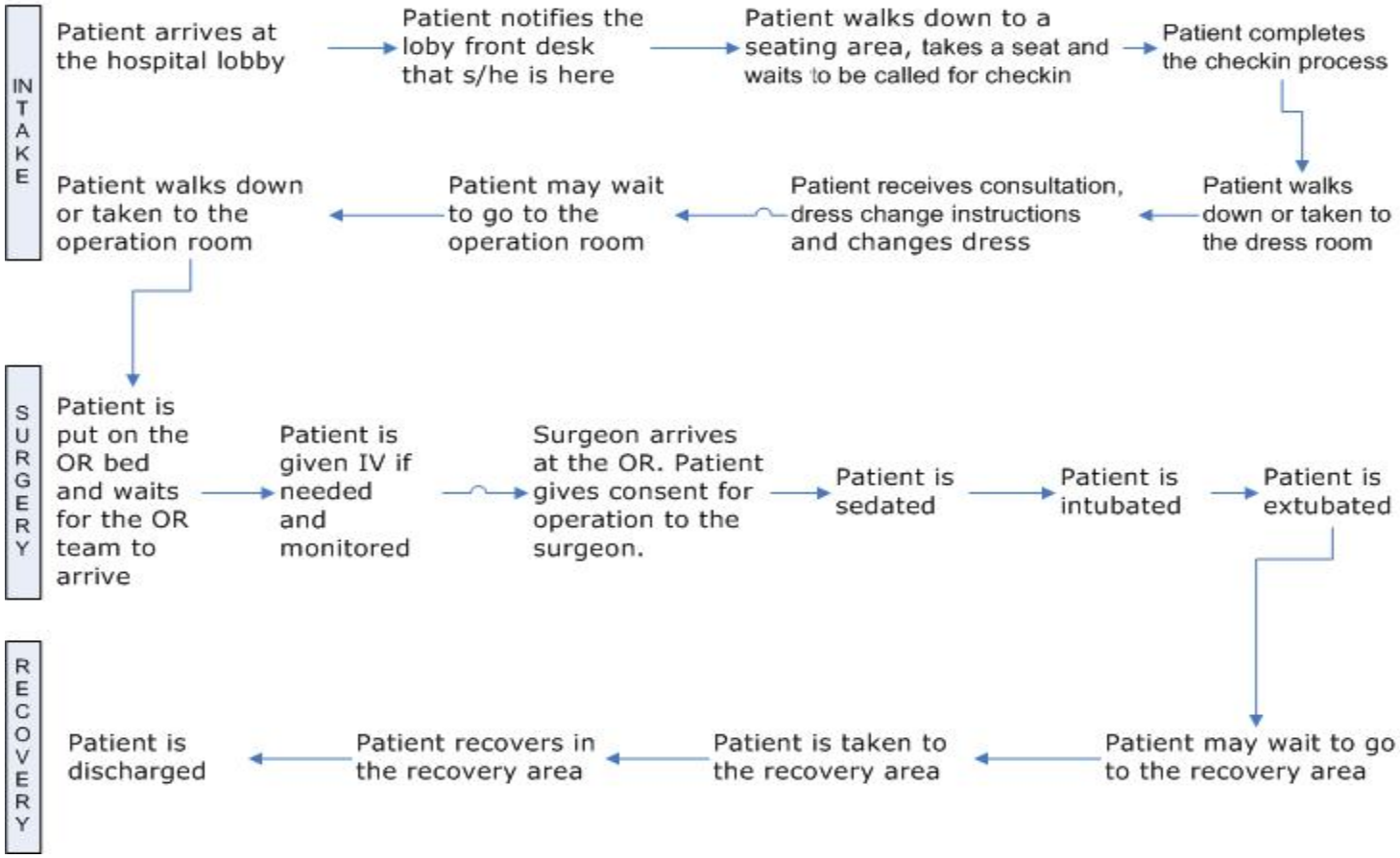
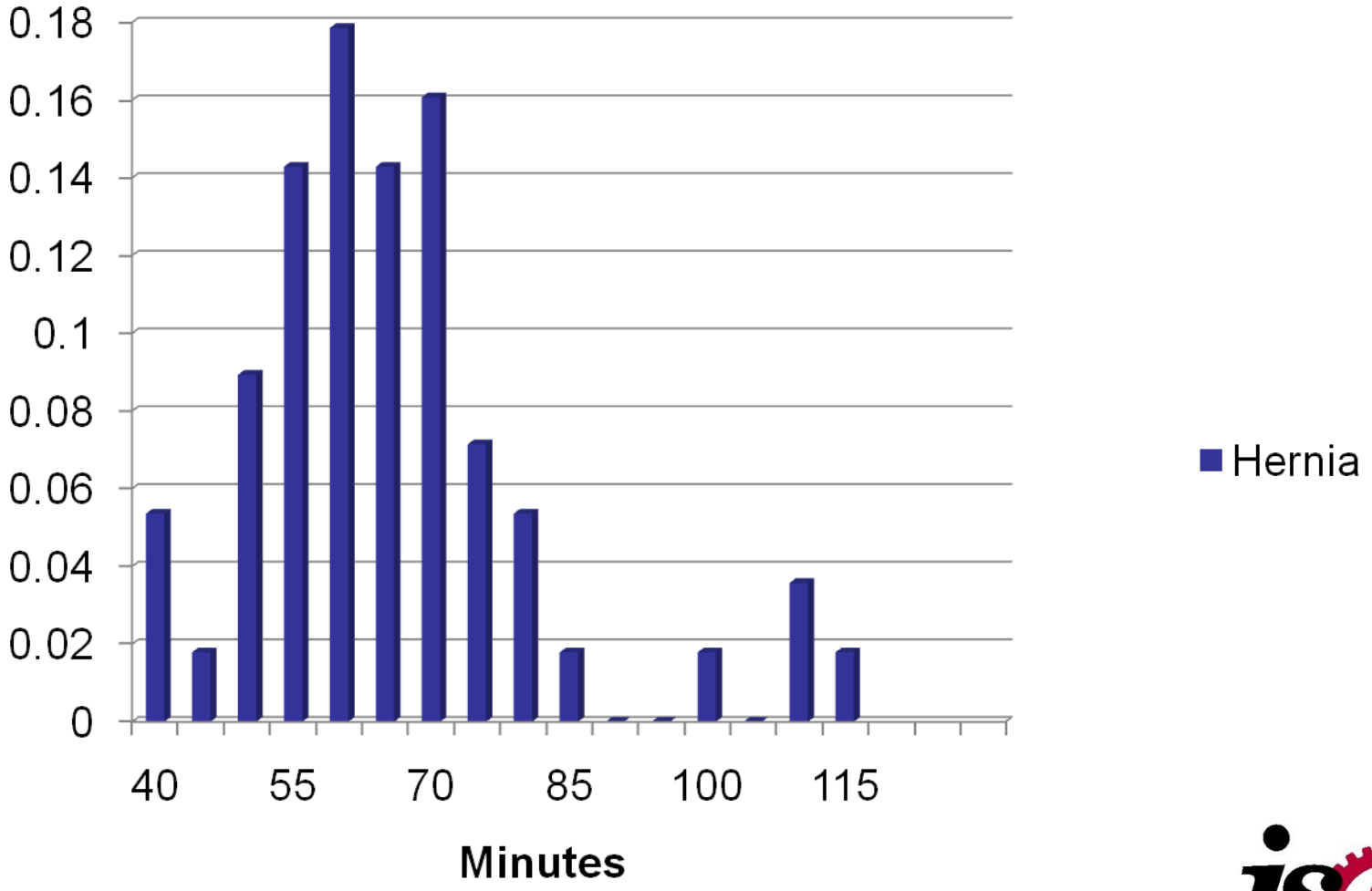
Outpatient Suite Process Map



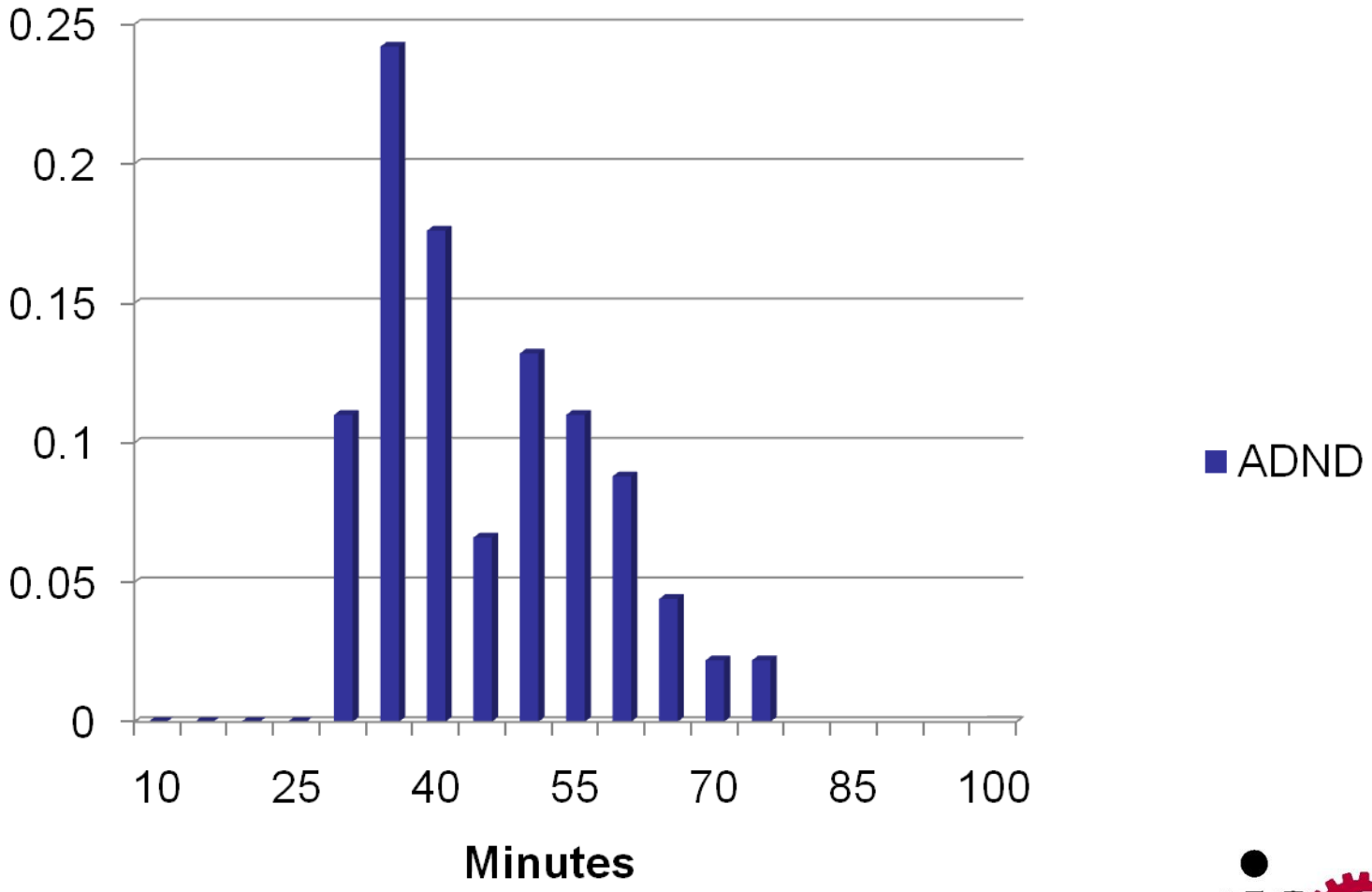
Complicating Factors

- Many types of resources to be scheduled: OR team, equipment, materials
- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Many competing criteria

Surgery Duration Uncertainty

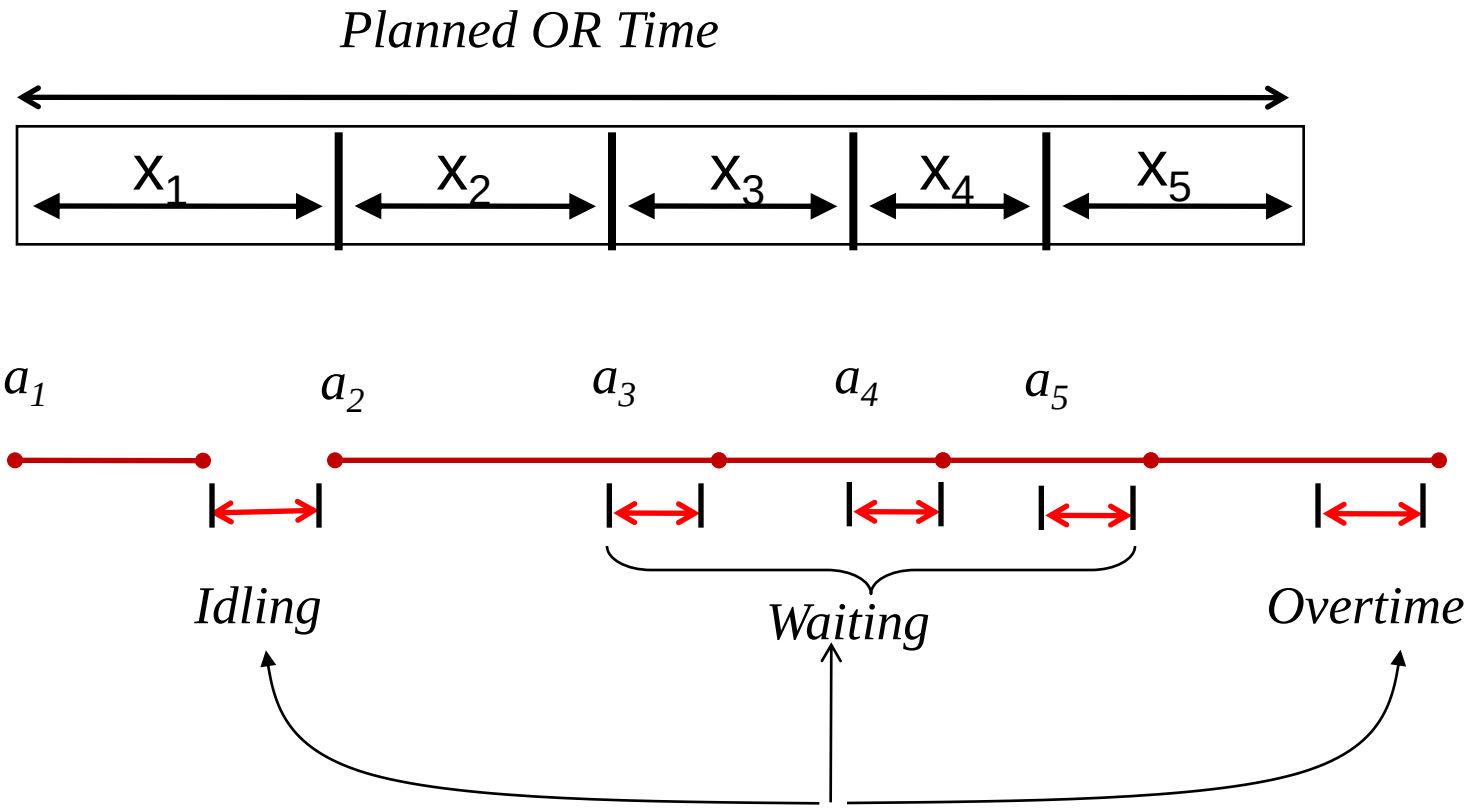


Surgery Duration Uncertainty



Problem 1: Single OR Scheduling

Single OR Scheduling - S(n)/G(n)/1



Min{ Idling + Waiting + Overtime}

Stochastic Optimization Model

$$\min\{\underbrace{\sum_{i=1}^{n} C_i^w * E_{\mathbf{Z}}[W_i]}_{\text{Cost of Waiting}} + \underbrace{\sum_{i=1}^{n} C^s * E_{\mathbf{Z}}[S_i]}_{\text{Cost of Idling}} + \underbrace{C^L * E_{\mathbf{Z}}[L]}_{\text{Cost of Overtime}}\}$$

$$W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)$$

$$S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)$$

$$L = \max(W_n + Z_n + \Sigma x_i - d, 0)$$

Literature Review – Single Server

- Queuing Analysis:
  - Mercer (1960, 1973)
  - Jansson (1966)
  - Brahimi and Worthington (1991)
- Heuristics:
  - White and Pike (1964)
  - Soriano (1966)
  - Ho and Lau (1992)
- Optimization:
  - Weiss (1990) – 2 surgery news vendor model
  - Wang (1993) – Exploited phase type distribution property
  - Denton and Gupta (2003) – General 2-SLP formulation

Stochastic Linear Program

$$\min\{E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\}$$

s.t.

$$\begin{aligned}
w_2 \quad - s_2 \quad &= Z_1 - x_1 \\
-w_2 + w_3 \quad - s_3 \quad &= Z_2 - x_2 \\
\ddots \qquad \ddots \quad & \\
-w_n \quad - s_n + l - g &= Z_n - d + \sum_{j=1}^{n-1} x_i
\end{aligned}$$

$$x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1,\ldots,n, \quad l, g \geq 0$$

Two Stage Recourse Problem

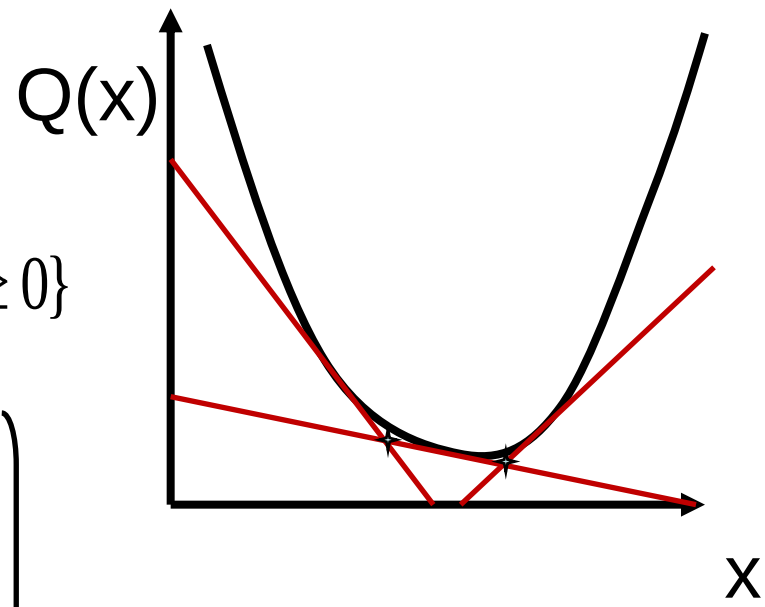
Initial Decision (x) → Uncertainty Resolved → Recourse (y)

$$\min\{Q(\mathbf{x}) = E_{\mathbf{Z}}[Q(\mathbf{x},\mathbf{Z})]\}$$

$$Q(\mathbf{x}) = \sum_{k=1}^{K} p^k Q(\mathbf{x},\mathbf{Z}^k)$$

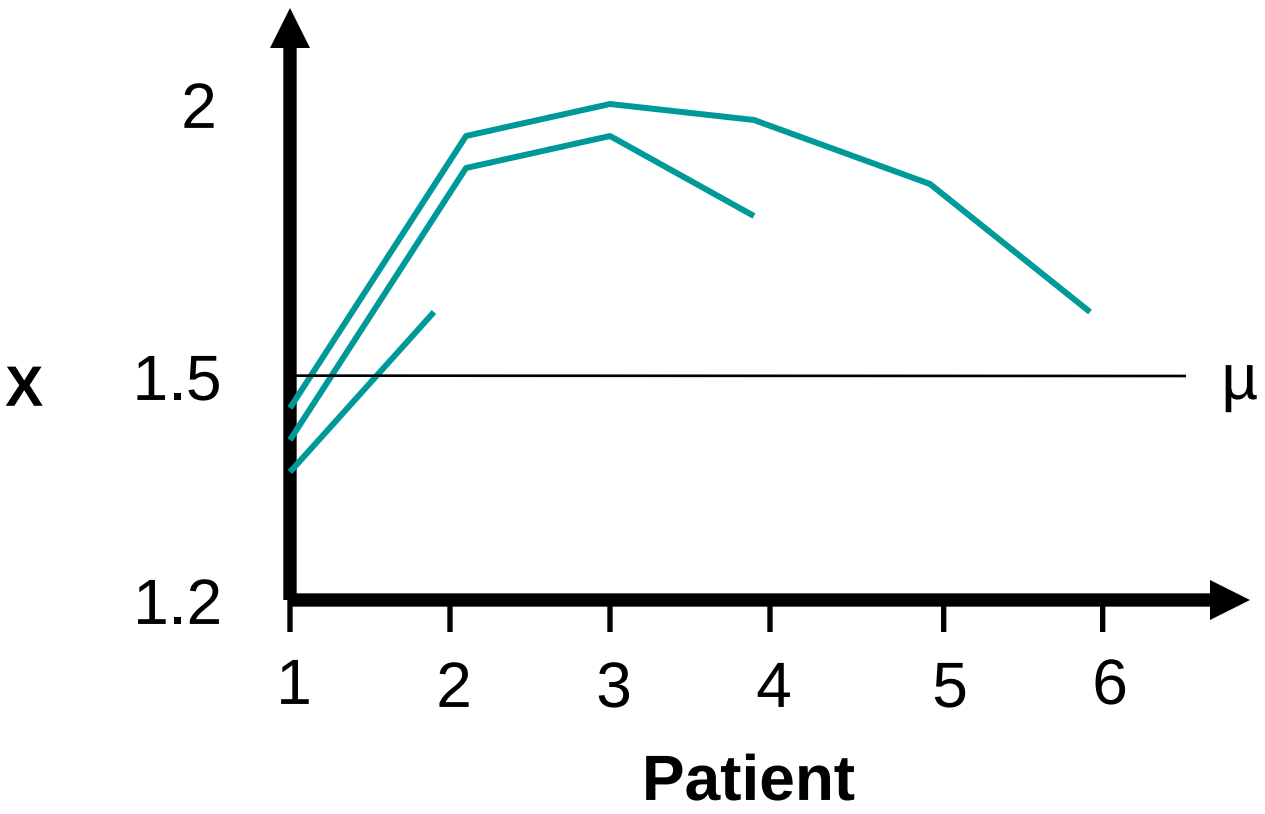
$$Q(\mathbf{x},\mathbf{Z}^k) = \min\{\mathbf{c}\cdot\mathbf{y}^k \mid \underbrace{T\,\mathbf{x} + W\,\mathbf{y}^k}_{} = \mathbf{h}^k, \mathbf{y}^k \geq 0\}$$

$$\begin{pmatrix} T & W^1 & & & \\ T & & W^2 & & \\ T & & & W^3 & \\ \vdots & & & & \ddots \\ T & & & & & W^K \end{pmatrix}$$



Example

- Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with U(1,2):



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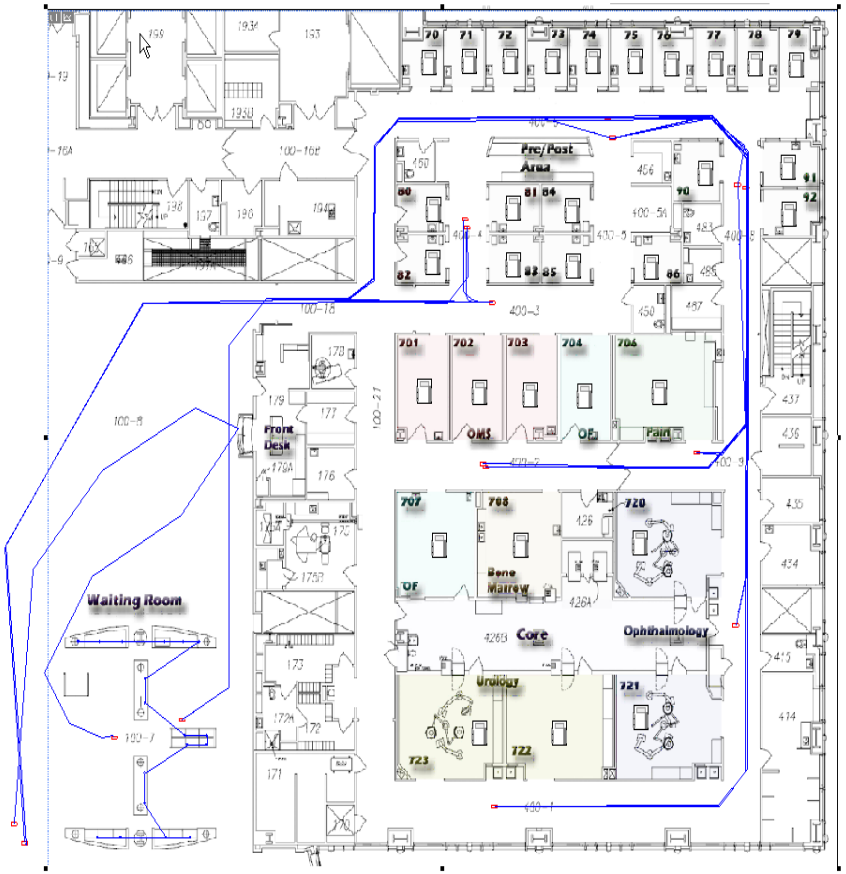
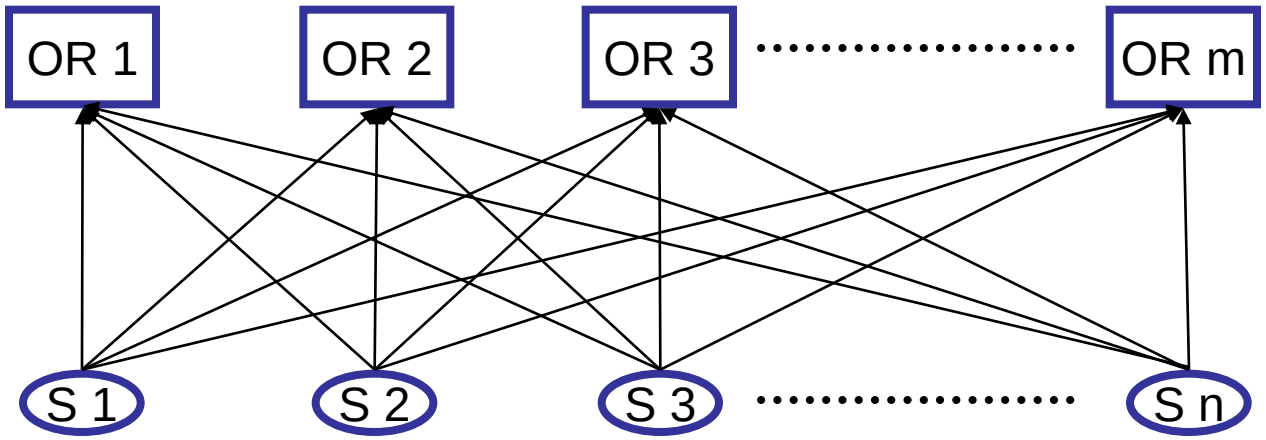
Surgical Suite Decisions

- Number of cases to schedule
- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling of staff and patients in intake, surgery, and recovery
- How to design the suite (intake rooms, recovery rooms, ORs)
- Selection of equipment resources (surgical kits, diagnostic equipment)



Problem 2: Multi-OR Surgery Allocation

Multi-Operating Room Scheduling



Decisions:

- How many operating rooms (ORs) to open?
- Which OR to schedule each surgery block in?

Performance Measures:

- Cost of operating rooms opened
- Overtime costs for operating rooms

Extensible Bin Packing

$$x_j = \begin{cases} 1 & if\ OR\ \ j\ open \\ 0 & if\ \ OR\ \ j\ closed \end{cases} \qquad y_{ij} = \begin{cases} 1 & if\ Surg.Block\ y\ i\ assigned\ to\ OR\ \ j \\ 0 & Otherwise \end{cases}$$

$$Z = \min\{\sum_{j=1}^{m} c^f x_j + c^v o_j\}$$

$$s.t. \qquad y_{ij} \le x_j \quad \forall(i,j)$$

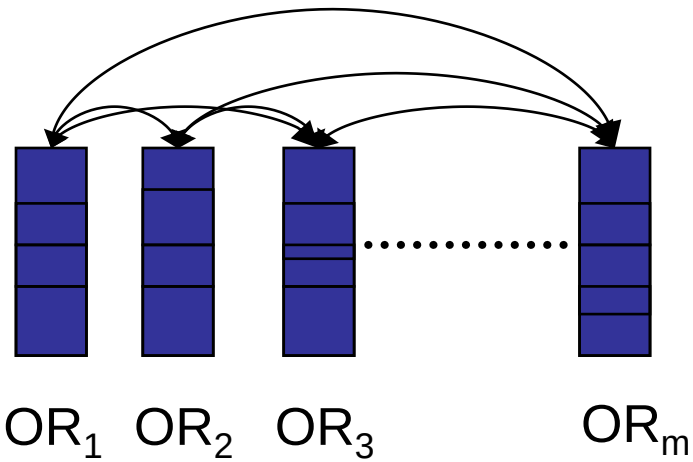
$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall(i)$$

$$\sum_{i=1}^{n} \bar{z}_i y_{ij} - o_j \le d_j x_j \quad \forall(i,j)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j \ge 0$$

Symmetry

- m! optimal solutions:



- Anti-symmetry constraints:

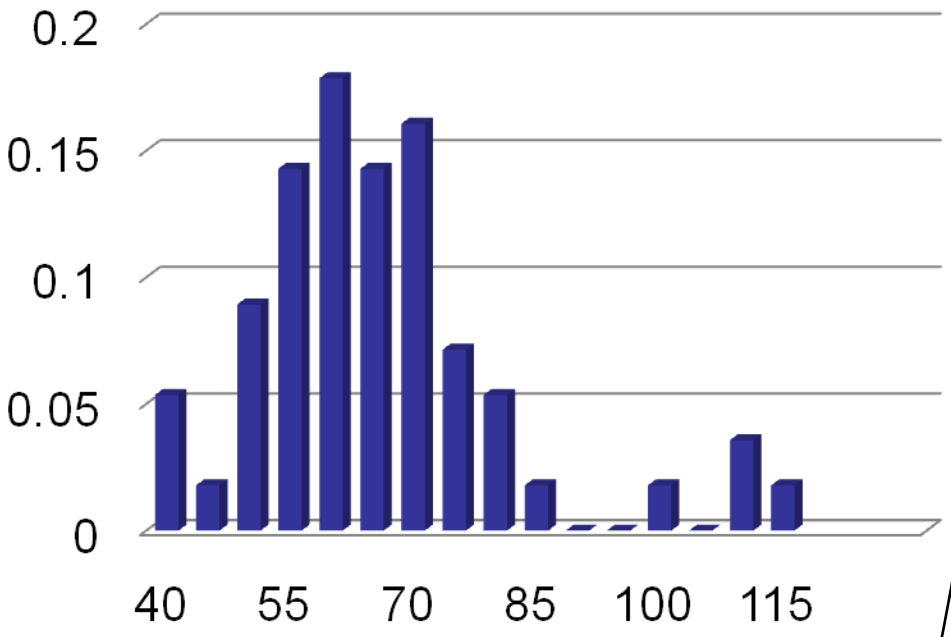
$$x_1 \geq x_2$$
$$x_2 \geq x_3$$
$$\vdots$$
$$x_m \geq x_{m-1}$$

OR Ordering

$$y_{11} = 1$$
$$y_{21} + y_{22} = 1$$
$$\vdots$$
$$\sum_{j=1}^{m} y_{mj} = 1$$

Surgery Assignment

Two-Stage Stochastic MIP



$$Q(\mathbf{x}) = \min\{\sum_{j=1}^{m} c^f x_j + c^v E_\omega[o_j(\omega)]\}$$

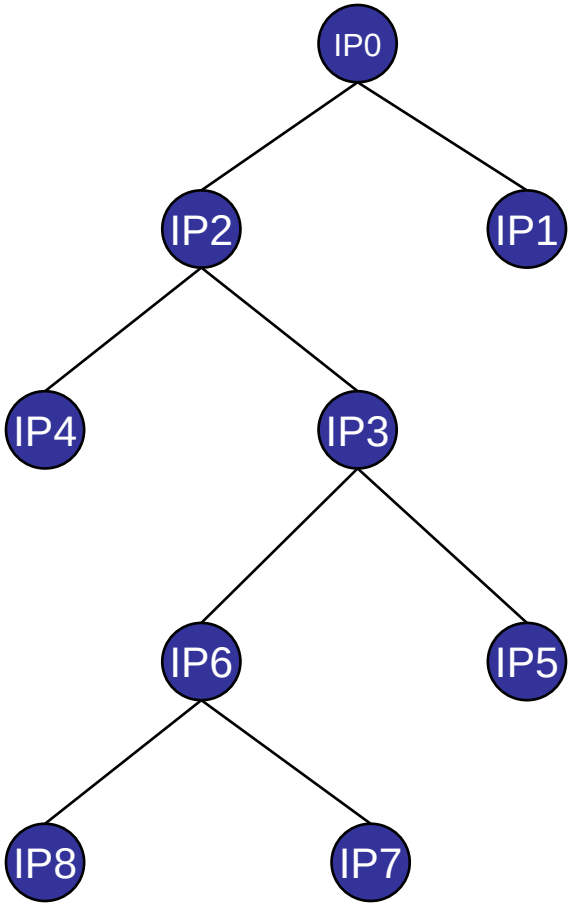
$$s.t. \quad y_{ij} \le x_j \quad \forall(i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall(i)$$

$$\sum_{i=1}^{n} Z_i(\omega) y_{ij} - o_j(\omega) \le d\ x_j \quad \forall(i,j,\omega)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \ge 0, \forall \omega$$

Integer L-Shaped Method



Master Problem:

$$Z = \min\{\sum_{j=1}^{m} c^f x_j + \Theta\}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall(i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall(i)$$

$$y_{ij}, x_j \in \{0,1\}, \Theta \ge 0$$

$$\Theta \ge E_\omega[\pi(h - Tx)]$$

Heuristic and Bounds

Dell'Ollmo (1998) – 13/12 approximation algorithm for bin packing with extensible bins

EBP Heuristic:

$$n \leftarrow LB;$$

$$LB = \left\lceil \frac{\sum_{i=1}^{n} \bar{z}_i}{T(1+\frac{c^f}{c^v T})} \right\rceil$$

$$repeat;$$

$$LPT(n);$$

$$if\,(o_j = 0, \forall j)\ Stop;$$

$$n \leftarrow n+1;$$

$$end(repeat);$$

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room

Robust Formulation

$$Z = \min\{\sum_{j=1}^{m} c^f x_j + F(x, y)\}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_j \in \{0,1\} \ge 0$$

$$F(x, y) = \begin{cases} \max_{\delta} \{\sum_{j=1}^{m} \eta_j\} \\ s.t. \quad \eta_j = c_j^v \max\{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - T_j x_j\}, \quad \forall j \\ \sum_{(i,j):y_{ij}=1}^{m} \frac{\delta_{ij} - \underline{z}_i}{\bar{z}_i - \underline{z}_i} y_{ij} \le \tau \\ \underline{z}_i \le \delta_{ij} \le \bar{z}_i, \forall (i, j) : y_{ij} = 1 \end{cases}$$

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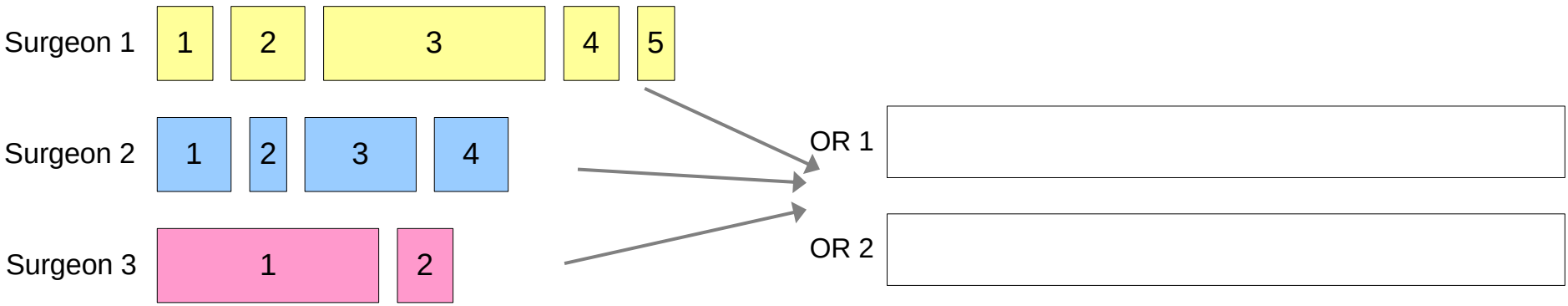
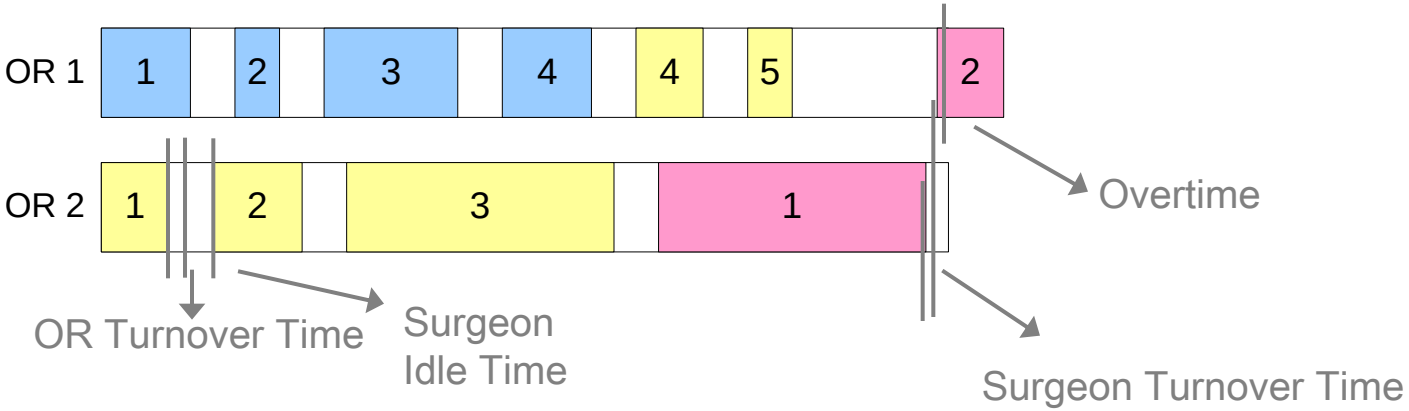
	15 surgery instances									
	Variable Cost = 0.033					Variable Cost = 0.0083				
			Robust IP					Robust IP		
Instance	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6
1	0.808	0.806	0.892	0.906	0.933	0.999	0.998	0.880	0.948	0.948
2	0.953	0.966	0.898	0.896	0.970	0.999	0.999	0.999	0.999	0.980
3	0.854	0.852	0.936	0.937	0.970	0.999	0.999	0.929	0.952	0.944
4	0.925	0.972	0.911	0.971	0.917	0.999	0.998	0.930	0.930	0.929
5	0.896	0.946	0.831	0.916	0.892	0.990	0.996	0.932	0.938	0.924
6	0.862	0.853	0.923	0.931	0.938	0.989	0.990	0.886	0.881	0.881
7	0.930	0.936	0.810	0.930	0.817	0.973	0.993	0.844	0.974	0.927
8	0.888	0.966	0.876	0.903	0.904	0.966	0.966	0.966	0.987	0.939
9	0.962	0.966	0.964	0.969	0.964	0.975	0.993	0.847	0.960	0.957
10	0.860	0.924	0.910	0.893	0.918	0.997	0.996	0.900	0.901	0.903
average	0.894	0.919	0.895	0.925	0.922	0.988	0.993	0.916	0.951	0.933
stdev	0.046	0.057	0.047	0.028	0.046	0.013	0.010	0.059	0.045	0.028

General Insights

- The fast LPT based heuristic works (fairly) well on a large number of instances
  - LPT works very well when overtime costs are low
  - LPT is better (and easier) than solving MV problem in most cases
- Robust IP is better than LPT when overtime costs are high

Current Research: Share ORs





Problem 3: Patient Arrival Scheduling

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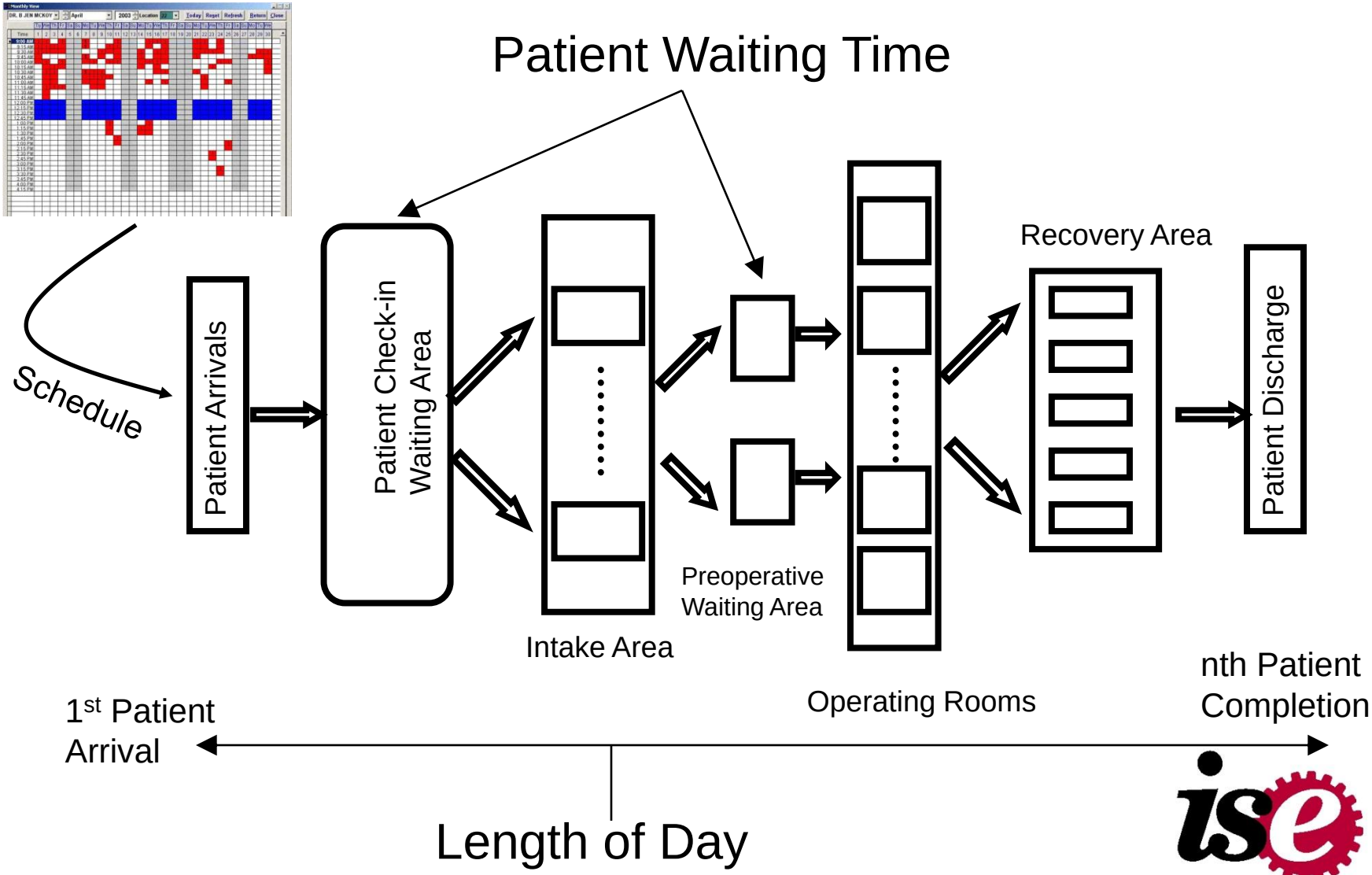
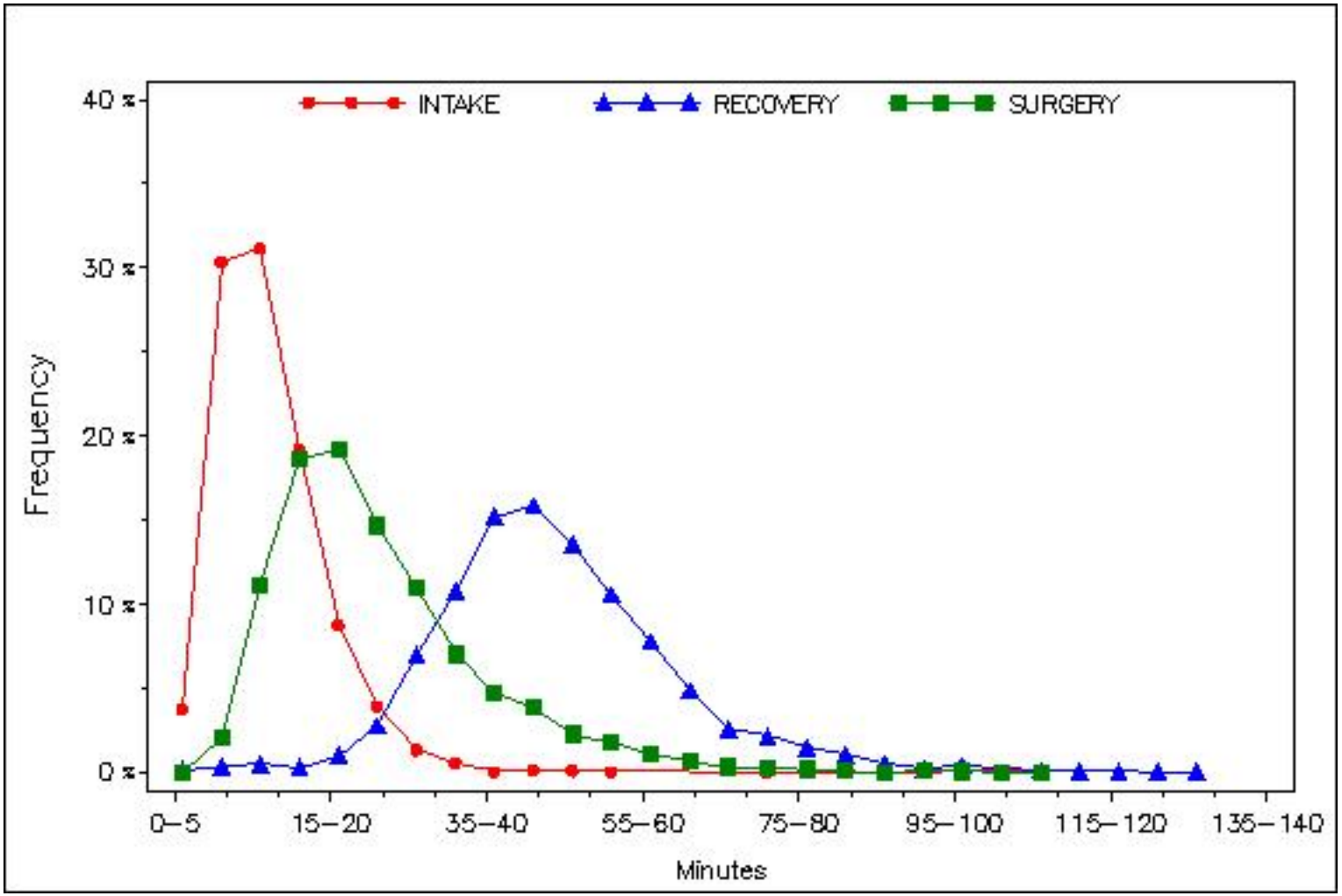
Endoscopy Suite



Intake, Surgery, and Recovery

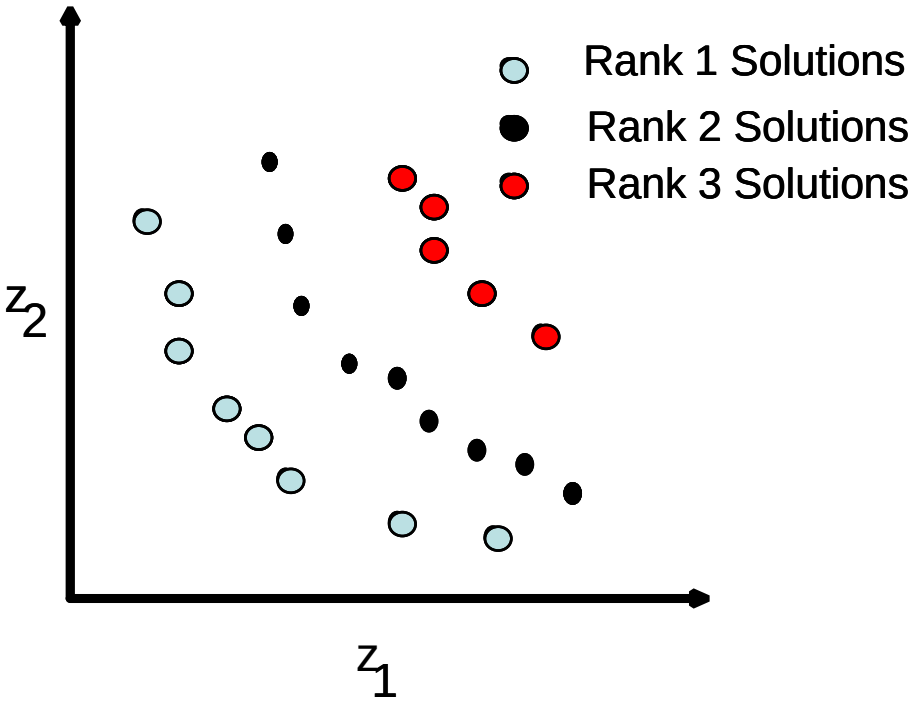


Simulation-optimization

- Decision variables: scheduled start times to be assigned to *n* patients each day
- Goal: Generate the set of non-dominated schedules to understand tradeoffs between waiting and length of day
- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA

Pareto Set

- The non-dominated sorting genetic algorithm (NSGA-II) of Deb *et al.*(2000) is used in the simulation optimization.



Selection Procedure

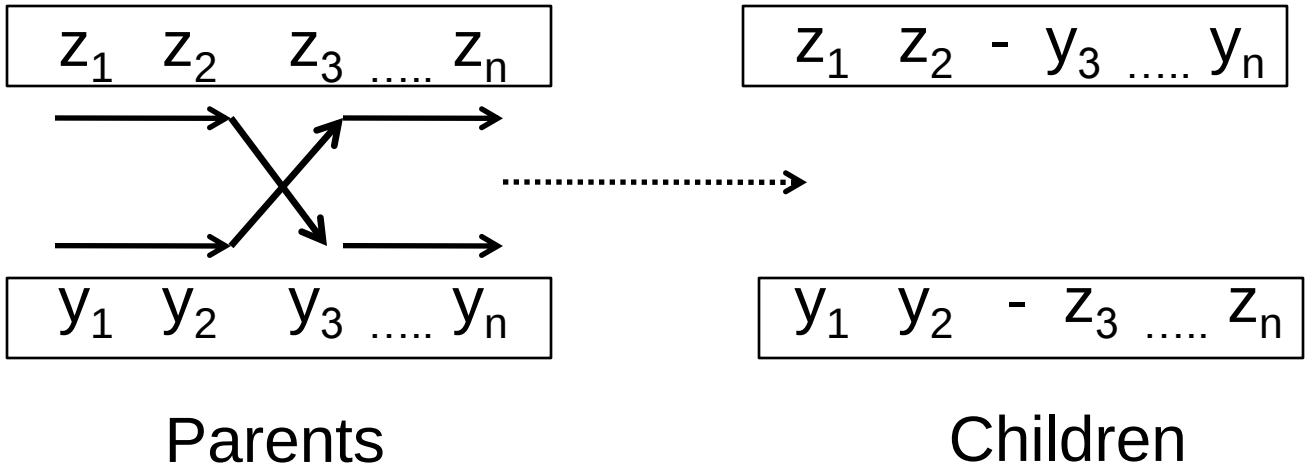
- Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) is used to compute the number of samples necessary to determine whether a solution *i* "dominates" *j*

- Solution *i* "dominates" *j* if:

$$E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]$$

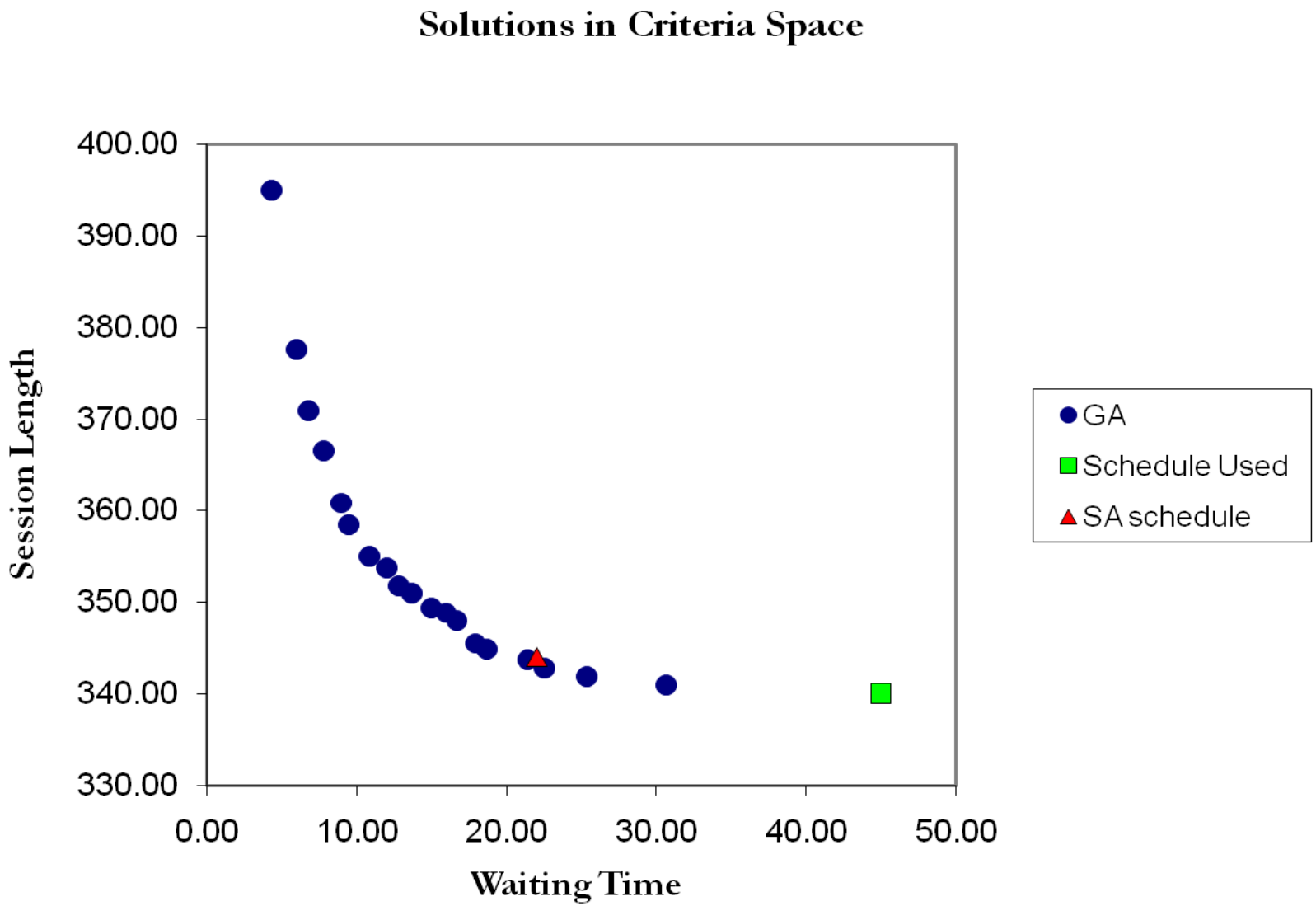
Genetic Algorithm

- Main features of the GA:
  - Randomly generated initial population of schedules
  - Selection based on 1) ranks and 2) crowding distance
  - Single point crossover:



  - Mutation

Example


Solutions in Criteria Space
Session Length
400.00
390.00
380.00
370.00
360.00
350.00
340.00
330.00
0.00
10.00
20.00
30.00
40.00
50.00
Waiting Time
GA
Schedule Used
SA schedule

Current and Future Research

- Investigating new stochastic programming and robust optimization formulations and methods
- Dynamic (online) scheduling problems
- Surgical suite design and re-configuration
- Real time control on the day of surgery

Questions?