NC STATE UNIVERSITY

Optimization of Health Care Delivery Systems

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Summary

- Health Care Operations:
  - Planning and scheduling of appointment-based service systems
  - Stochastic Programming and Robust Optimization
- Medical Decision Making:
  - Screening and treatment decisions for chronic diseases
  - Markov decision processes (MDPs) and Partially observable MDPs

Appointment-Based Service Systems

Collaborators

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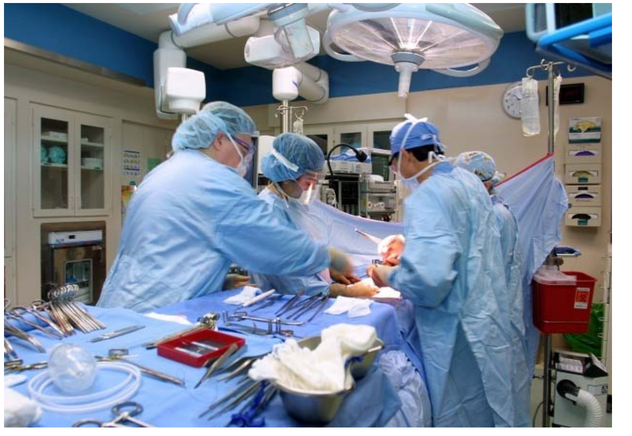
Summary

- Optimization models for appointment based stochastic service systems
  - Single server scheduling
  - Multiple server scheduling
  - Dynamic scheduling
  - Bi-criteria scheduling
  - Scheduled service networks

Surgery

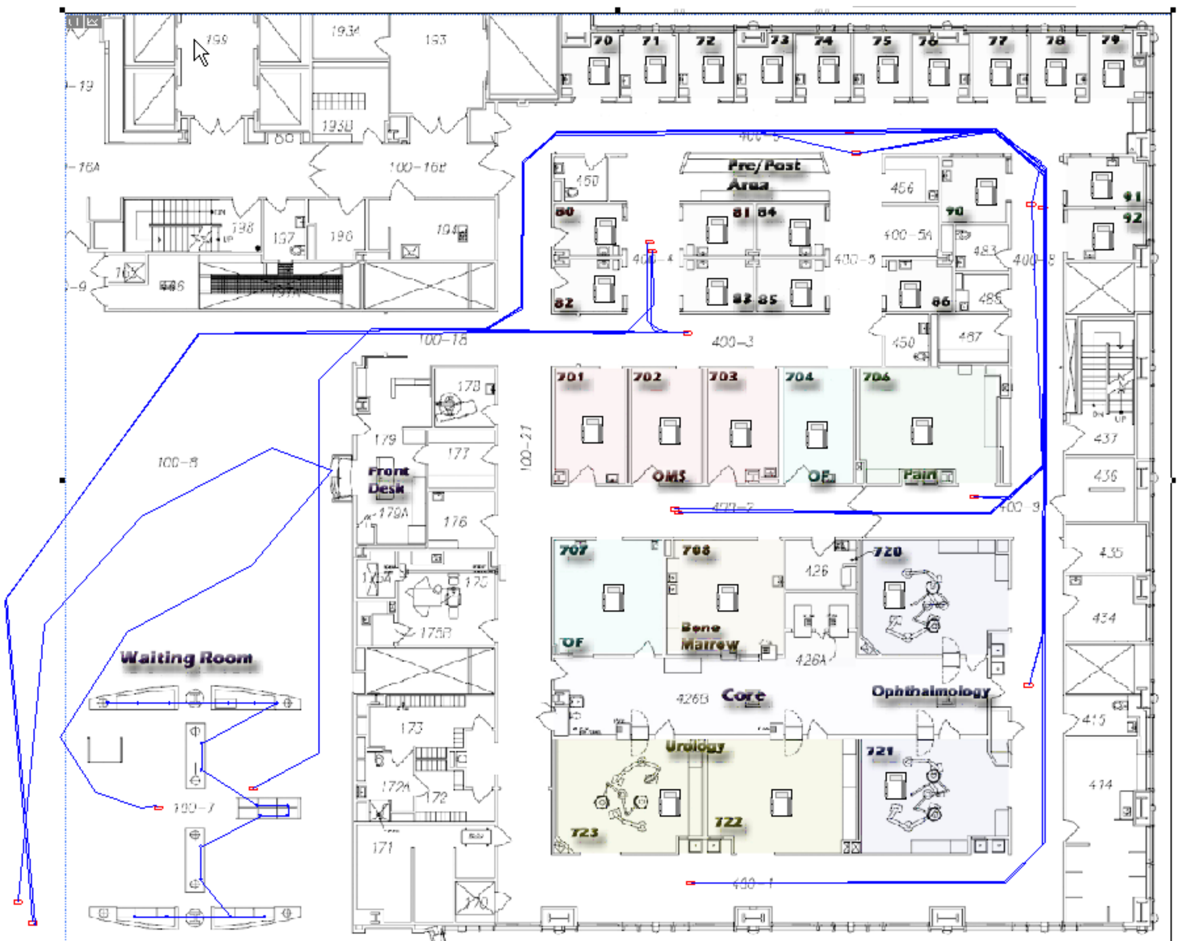
- <u>Patient Intake:</u> administrative activities, pre-surgery exam, gowning, site prep, anesthetic
- <u>Surgery:</u> incision, one or multiple procedures, pathology, closing
- <u>Recovery:</u> post-anesthesia care unit (PACU), ICU, hospital bed



Complicating Factors

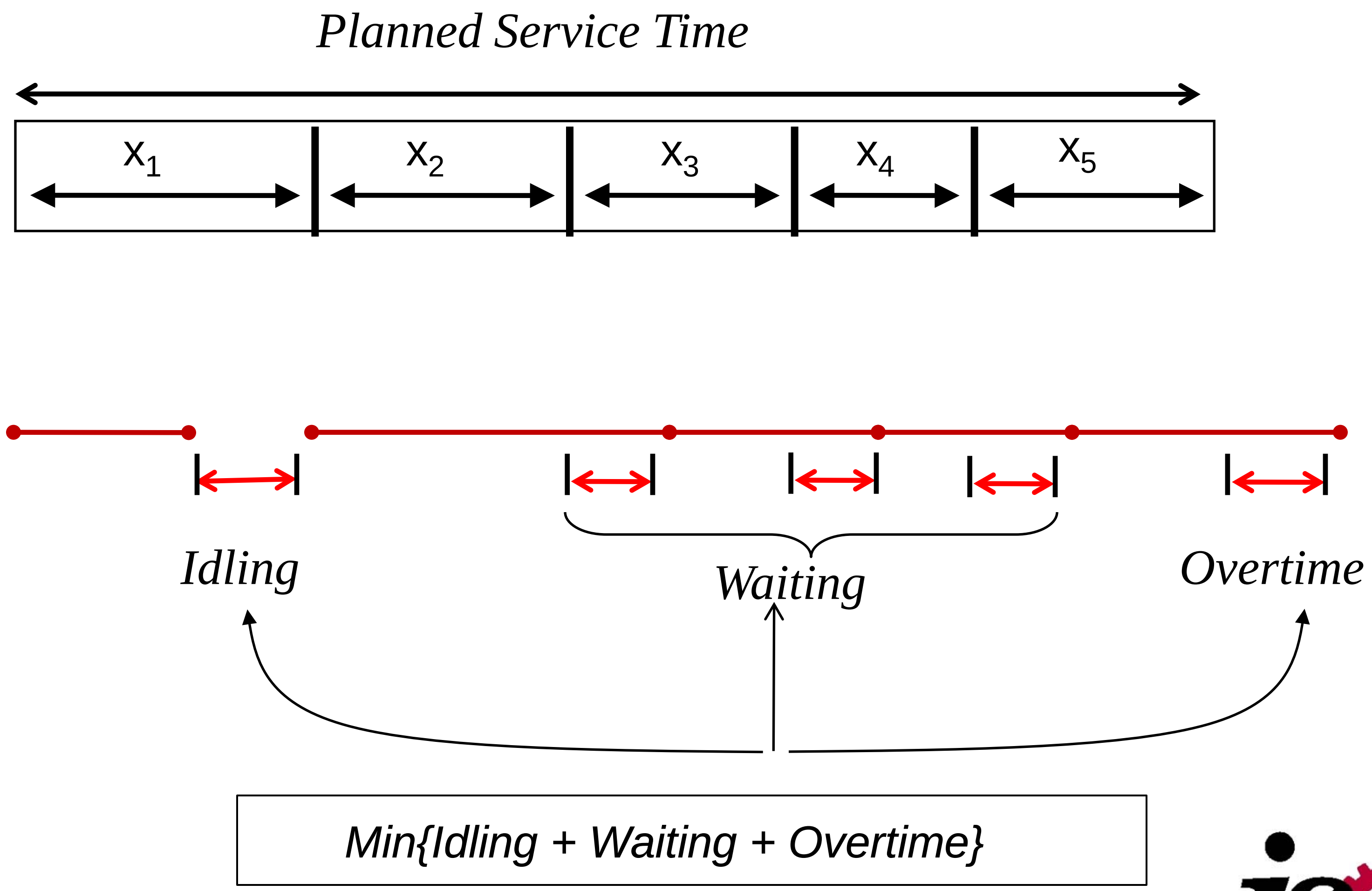
- Multiple expensive resources
- Large number of activities to be completed in a fixed period of time
- Uncertainty in duration of activities
- Many competing criteria

Outpatient Procedure Center



Single Server Scheduling

Single OR Scheduling (S(n)/G(n)/1)



Stochastic Optimization Model

$$\min\{\underbrace{\sum_{i=1}^{n} c_i^w E_{\mathbf{Z}}[w_i]}_{\text{Cost of Waiting}} + \underbrace{\sum_{i=1}^{n} c^s E_{\mathbf{Z}}[s_i]}_{\text{Cost of Idling}} + \underbrace{c^L E_{\mathbf{Z}}[l]}_{\text{Cost of Overtime}}\}$$

$$w_i = \max(w_{i-1} + Z_{i-1} - x_{i-1}, 0),\ i = 1,..,n-1$$

$$s_i = \max(-w_{i-1} - Z_{i-1} + x_{i-1}, 0),\ i = 1,...,n-1$$

$$l = \max(w_n + Z_n + \sum_{i=1}^{n-1} x_i - d, 0)$$

Stochastic Linear Program

$$\min\{E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\}$$

s.t.

$$\begin{array}{llllll}
w_2 & & - s_2 & & & = Z_1 - x_1 \\
-w_2 + w_3 & & & - s_3 & & = Z_2 - x_2 \\
 & \ddots & & \ddots & & \vdots \\
 & -w_n & & & - s_n + l - g & = Z_n - d + \sum_{j=1}^{n-1} x_i
\end{array}$$

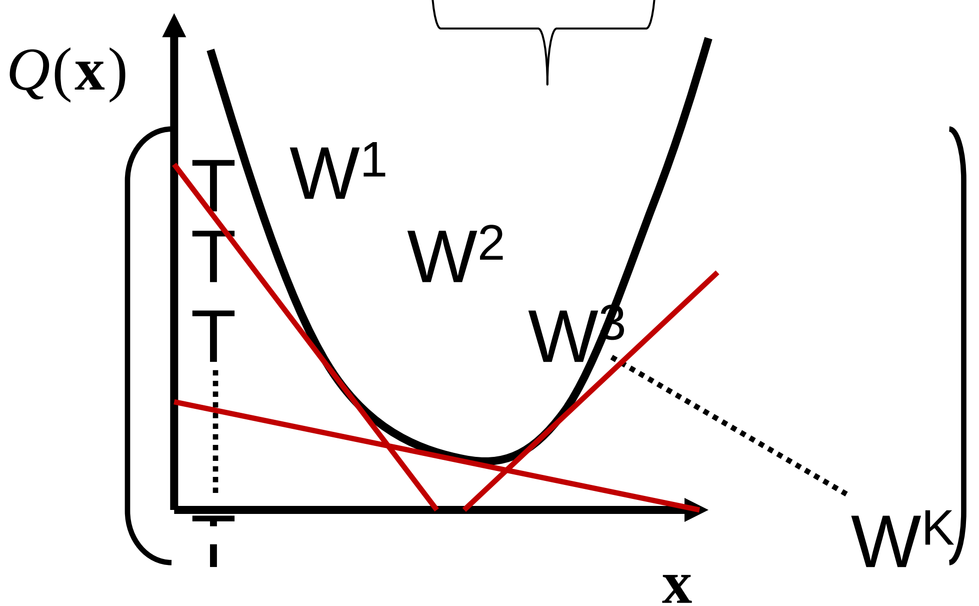
$$x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1,\ldots,n, \quad l, g \geq 0$$

Two Stage Recourse Problem

Initial Decision (x) → Uncertainty Resolved (Z) → Recourse (y)

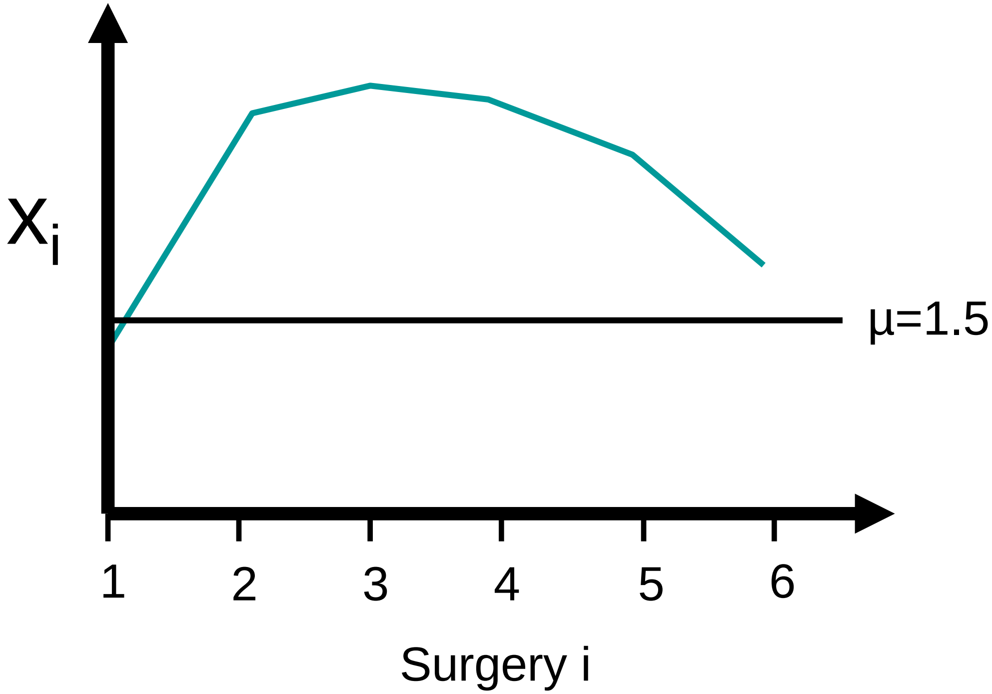
$$\min\{Q(\mathbf{x}) = E_{\mathbf{Z}}[Q(\mathbf{x},\mathbf{Z})]\}$$

$$Q(\mathbf{x},\mathbf{Z}^k) = \min\{\mathbf{c}\cdot\mathbf{y}^k \mid \underbrace{T\,\mathbf{x} + W\,\mathbf{y}^k = \mathbf{Z}^k}, \mathbf{y}^k \geq 0\}$$



Example

- Schedule for n= 7 with i.i.d. distributions with U(1,2):

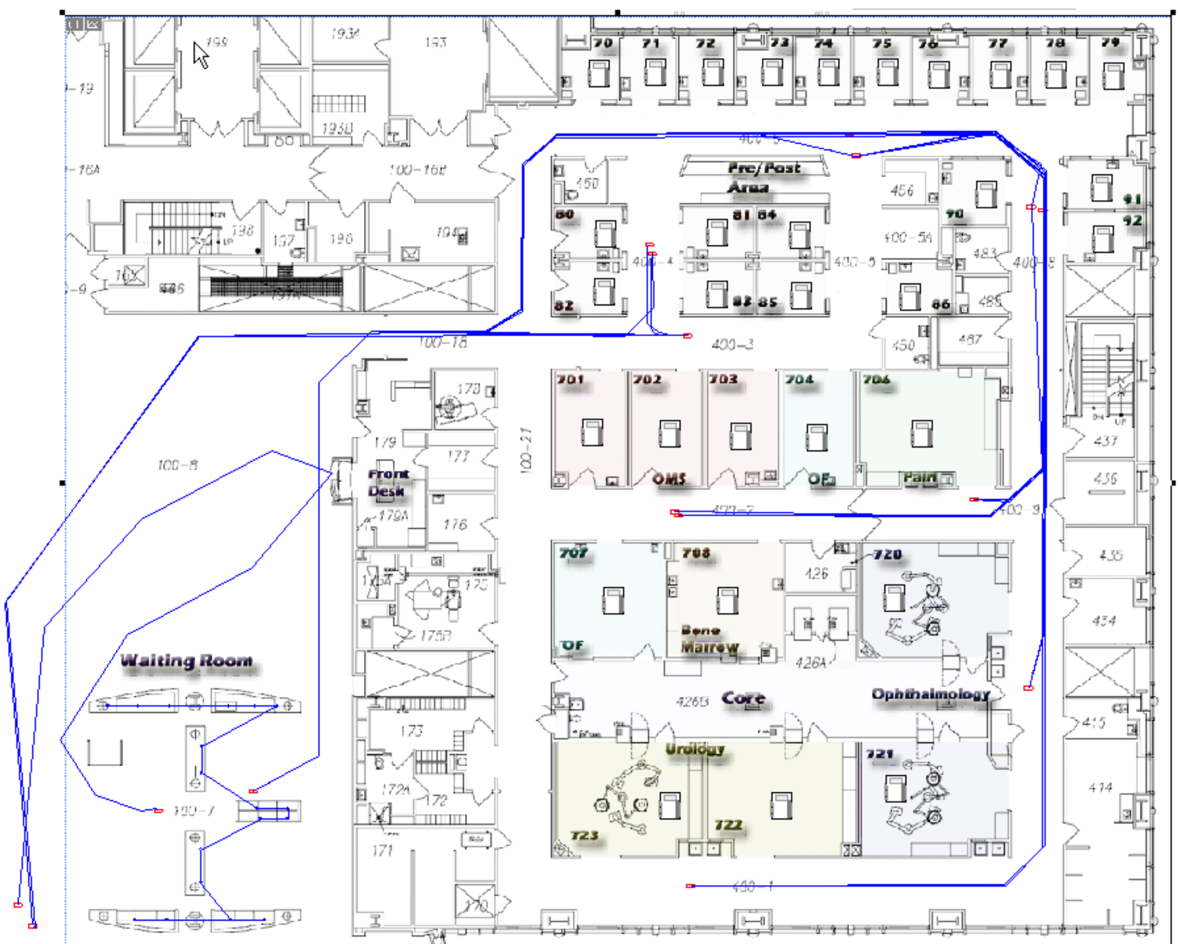


Denton, B.T., Gupta, D., "A Sequential Bounding Approach for Optimal Appointment Scheduling," *IIE Transactions*, 35, 1003-1016, 2003

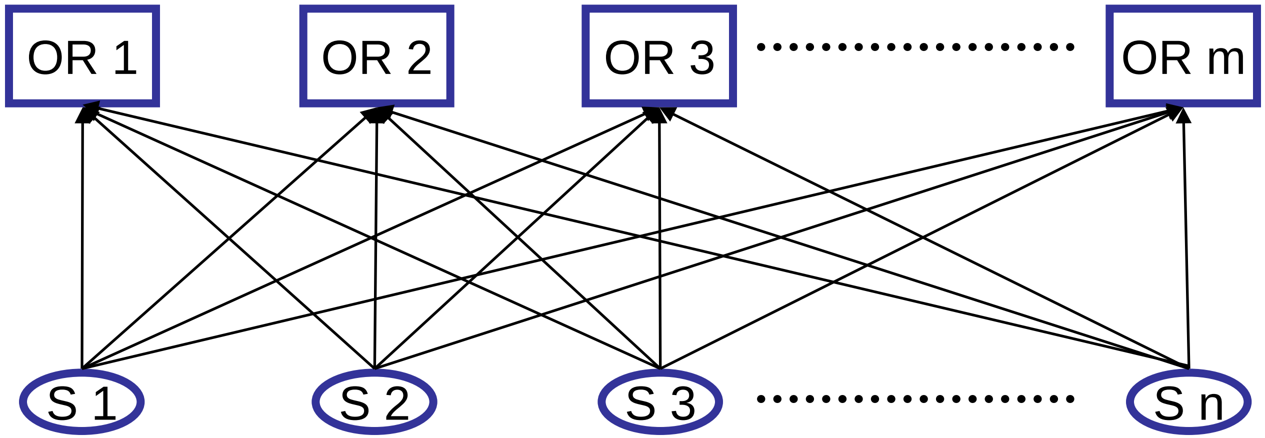
Denton, B.T., Viapiano, J, Vogl, A., "Optimization of Surgery Seqencing and Scheduling Decisions Under Uncertainty," *Health Care Management Science,* 10(1), 13-24, 2007

Multi-OR Surgery Allocation

Outpatient Procedure Center



Multi-OR Room Scheduling



Performance Measures:

- Cost of opening ORs
- Overtime costs

Decisions:

- How many ORs to open?
- Which OR to schedule each surgery block?

Extensible Bin Packing

$$x_j = \begin{cases} 1 \text{ if OR } j \text{ open} \\ 0 \text{ if OR } j \text{ closed} \end{cases} \qquad y_{ij} = \begin{cases} 1 \text{ if Surgery } i \text{ assigned to OR } j \\ 0 \quad \text{Otherwise} \end{cases}$$

$$Z = \min\{\sum_{j=1}^{m} c^f x_j + c^v o_j\}$$

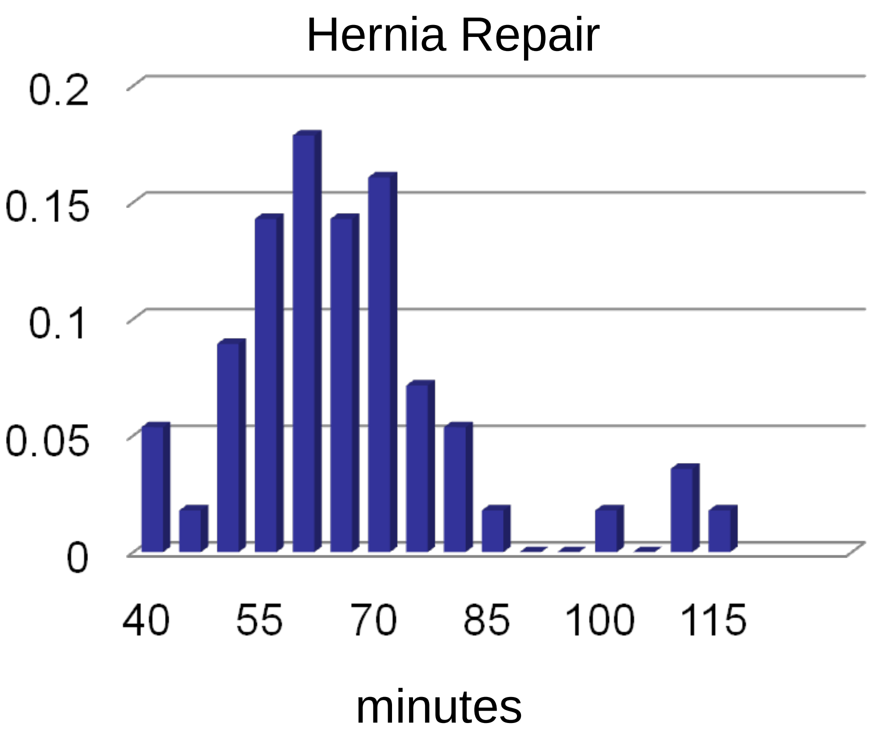
$$s.t. \qquad y_{ij} \le x_j \quad \forall (i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \ \forall (i)$$

$$\sum_{i=1}^{n} b_i y_{ij} - o_j \le d_j x_j \quad \forall (i,j)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j \ge 0$$

Two-Stage Stochastic MIP



$$Q(\mathbf{x}) = \min\{\sum_{j=1}^{m} c^f x_j + c^v E_\omega[o_j(\omega)]\}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall(i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \;\; \forall(i)$$

$$\sum_{i=1}^{n} b_i(\omega) y_{ij} - o_j(\omega) \le d_j x_j \quad \forall(i,j,\omega)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \ge 0, \forall \omega$$

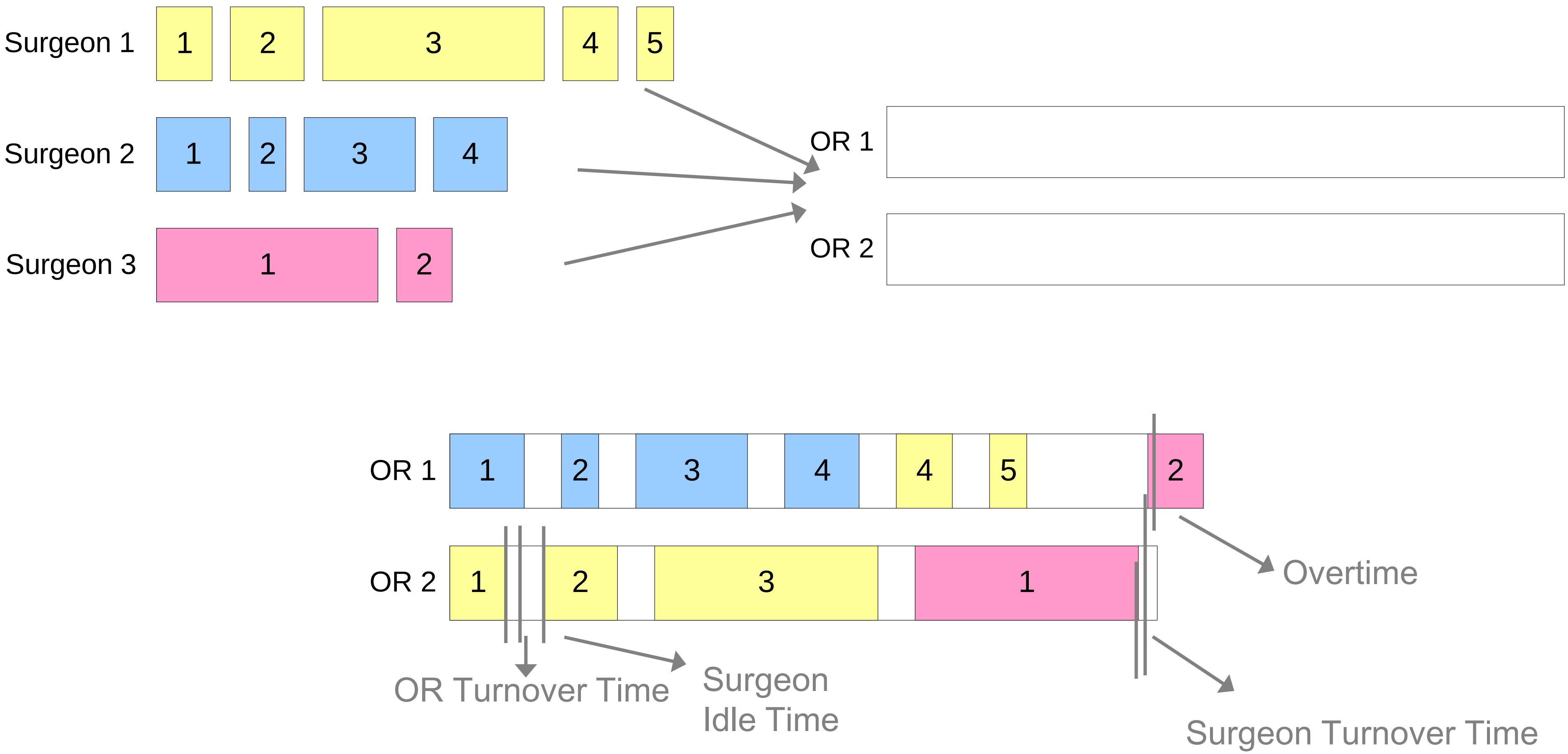
General Insights

- New upper and lower bounds on the extensible bin-packing problem
- Valid inequalities to reduce the impact of symmetry
- Decomposition based solution methods
- New Robust MIP formulation

Denton, B.T., Miller, A., Balasubramanian, H., Huschka, T., "Optimal Surgery Block Allocation Under Uncertainty," *Operations Research* (in press), 2009

Other Research Directions

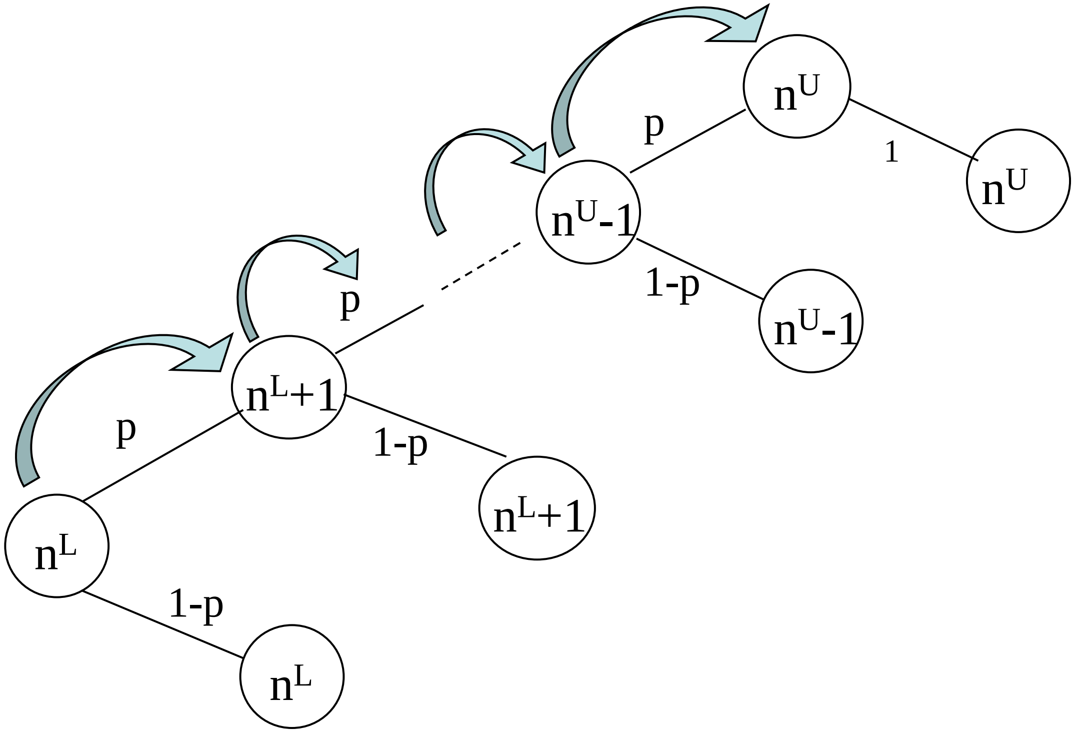
Pooling OR Capacity



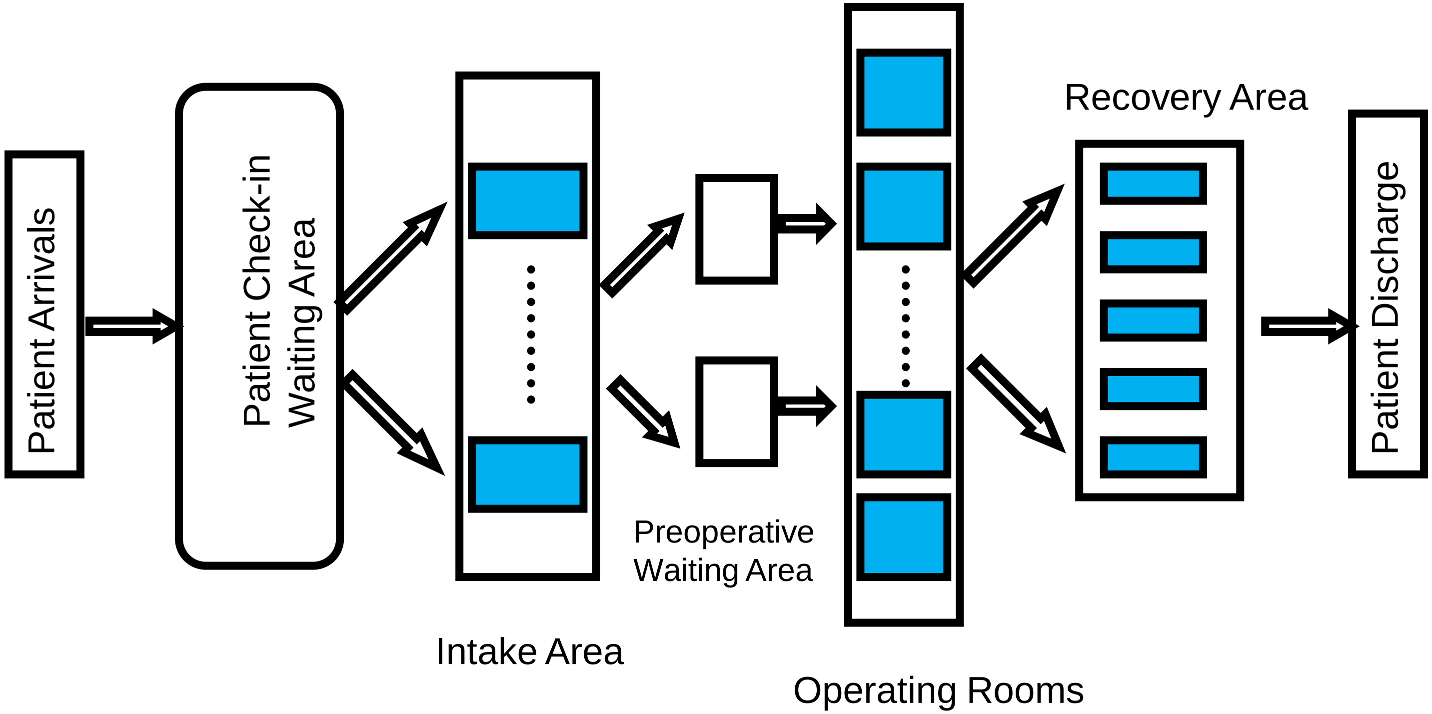
Batun, S., Denton, B.T., Huschka, T.R., Schaefer, A.J., “The Benefit of Pooling Operating Rooms Under Uncertainty,” Working Paper, 2009

Dynamic Appointment Scheduling

- Patients request appointments stochastically
- Appointment decisions are made one at a time
- Multistage stochastic linear program



Surgical Suite Scheduling

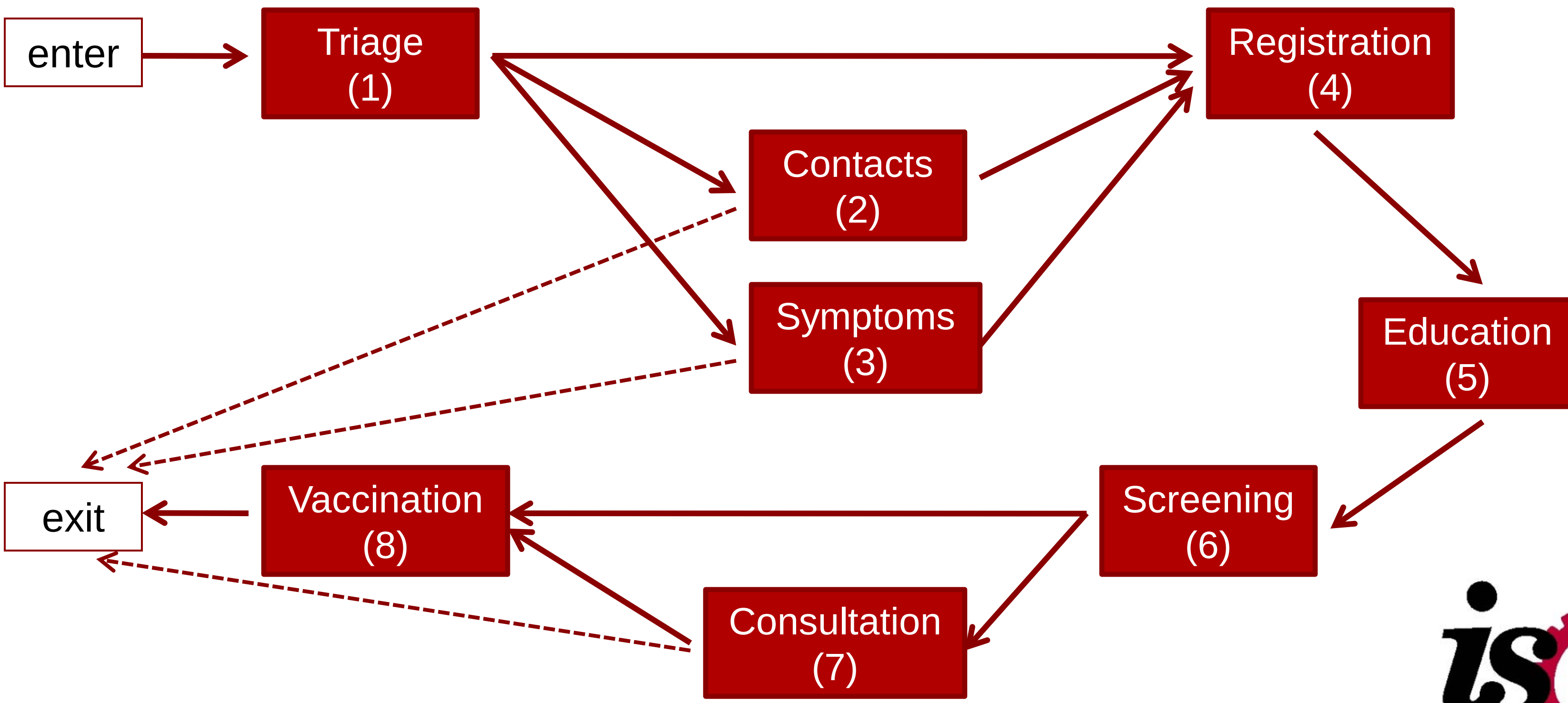


Berg, B., Denton, B.T., Nelson, H., Balasubramanian, B., Rahman, A., Bailey, A., Lindor, K., "A Computer Simulation Model to Evaluate Operational Performance of a Colonoscopy Suite," *Medical Decision Making* (in press), 2009

Gul, S., Denton, B.T., Huschka, T., Fowler, J.R., "Bi-criteria Evaluation of an Out Patient Surgery Clinic via Simulation," submitted to *POM*, December 2008

POD Scheduling

- POD Design from Aaby, et al. Montgomery County's Public Health Service Uses Operations Research to Plan Emergency Mass Dispensing and Vaccination Clinics. *Interfaces*, 36(6), pp. 569–579, 2006.



Chronic Disease Screening and Treatment

Collaborators

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Summary

- Disease screening under imperfect information
  - Biopsy referral decisions
  - National screening policies

- Optimal timing of medical treatment decisions
  - Treatment optimization
  - Adherence control

Disease Test Beds

- Type 2 Diabetes:
  - Mayo Clinic DEMS
  - Ingenix Inc.
- Prostate Cancer:
  - Olmsted county medical record
  - Mayo Clinic radical prostatectomy database
- Bladder cancer
  - UNC Medical Record
  - SEER

Diabetes

- There are more than 23 million people in the U.S. who have diabetes
  - 8% of the U.S. population
  - 90% have type 2 diabetes

- Two out of three people with diabetes will die from either stroke or coronary heart disease (CHD)

Statins

LESCOL XL
(fluvastatin sodium) 80 mg
extended-release tablets

CRESTOR
rosuvastatin calcium

Cost of Statins

- More than $20 billion dollars are spent annually in the U.S. alone
- Side effects including liver failure, muscle pain, dizziness, nausea, headaches…
- Currently there is broad disagreement about the best policy for statin treatment

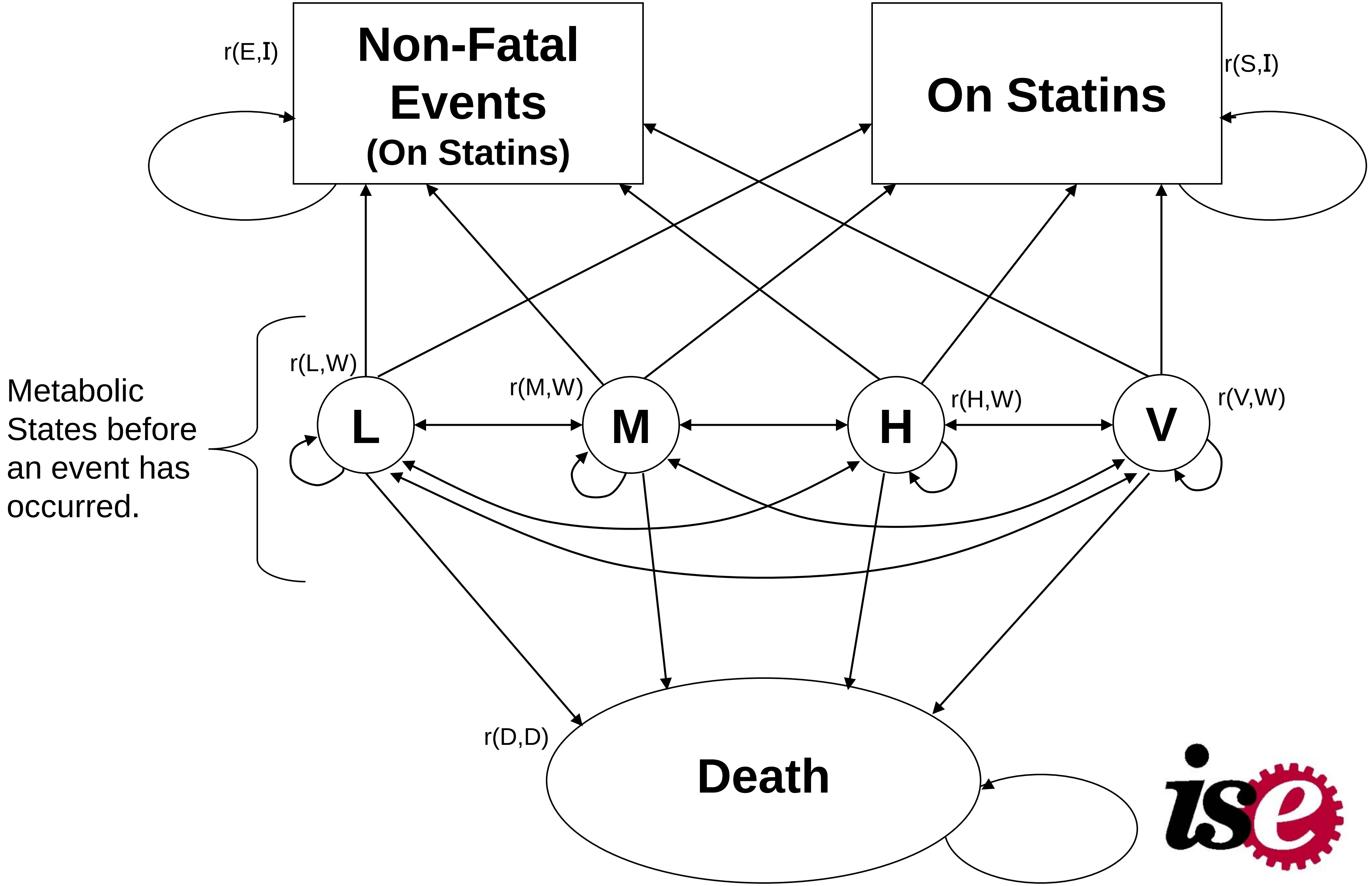
CHD & Stroke Risks

- What other factors affect CHD and Stroke risk?
  - Age
  - Gender
  - Ethnicity
  - Smoking
  - Blood Pressure
  - Hemoglobin A1c
  - Exercise & Diet
  - Body Mass Index
- In recent years several risk models have been developed to calculate the risk of CHD & Stroke:
  - UKPDS
  - Framingham
  - Archimedes

Markov Decision Process Model

- Stages:
  - Time horizon: Ages 40-100
  - Annual decision epochs
- Decision:
  - **Initiate** or **delay** statin treatment
- States:
  - Metabolic: Total cholesterol and HDL (each can be L, M, H, V), Blood pressure, HbA1c
  - Demographic: Gender, Race, BMI, smoking status, medical history

State Transition Diagram



Optimal Treatment Policy

- Society
  - Maximize a weighted combination of patient rewards for life years minus costs of treatment and health services
- Patient
  - Maximize rewards for quality adjusted life years
- Third-party Payer
  - Minimize costs of treatment and health services

Denton, B.T., Kurt, M., Shah, N.D., Bryant, S.C., Smith, S.A., "A Markov Decision Process for Optimizing the Start Time of Statin Therapy for Patients with Diabetes," *Medical Decision Making*, 29(3), 351-367, 2009

Model Formulation

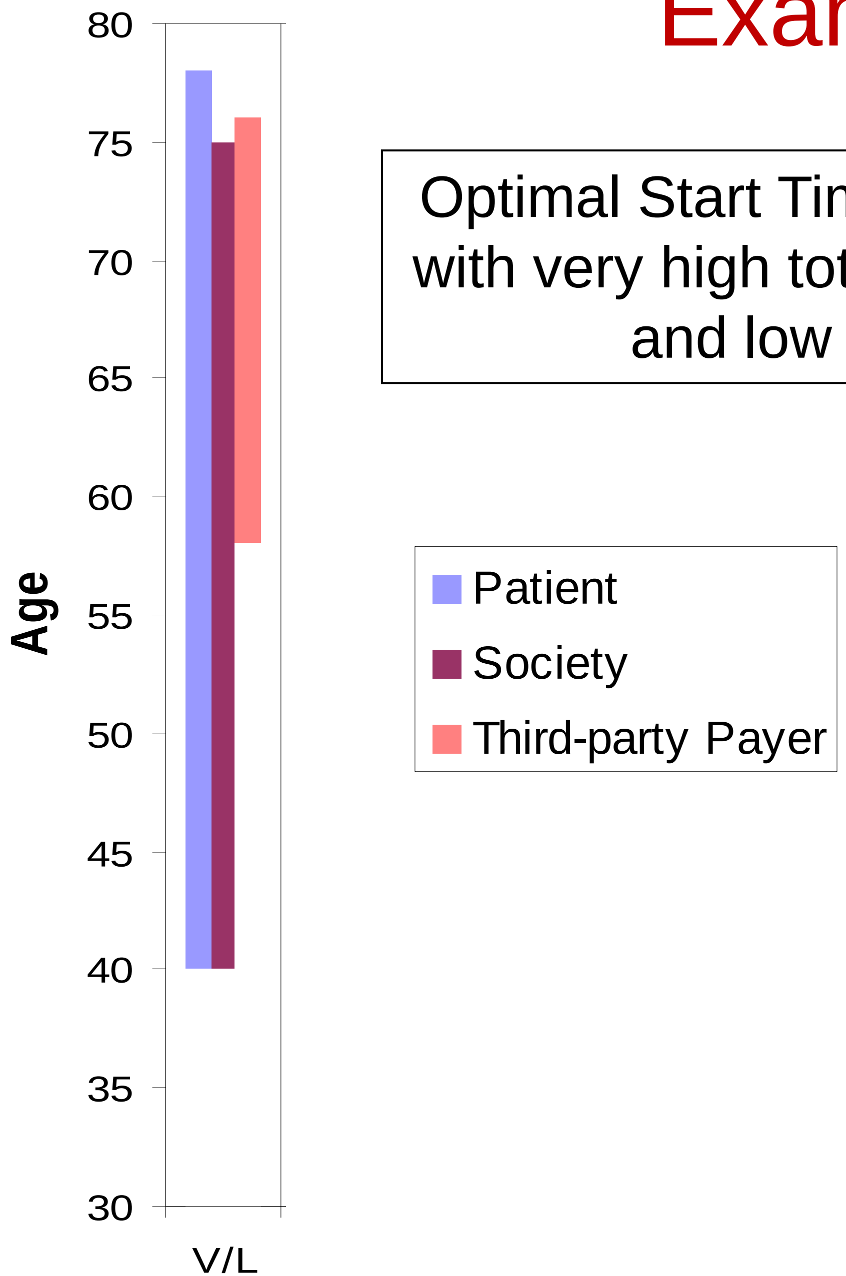
- Decision epochs: $t \in T = \{1,2,3,\ldots., N\}$
- Health States: $s_t \in S = \{1,2,3,\ldots, L, L+1\}$
- Treatment Status: $m \in \{0,1\}$
- Action: $a_t(s_t) = \begin{cases} I, W & \text{if m} = 0 \\ W & \text{if m} = 1 \end{cases}$

Optimality Equations:

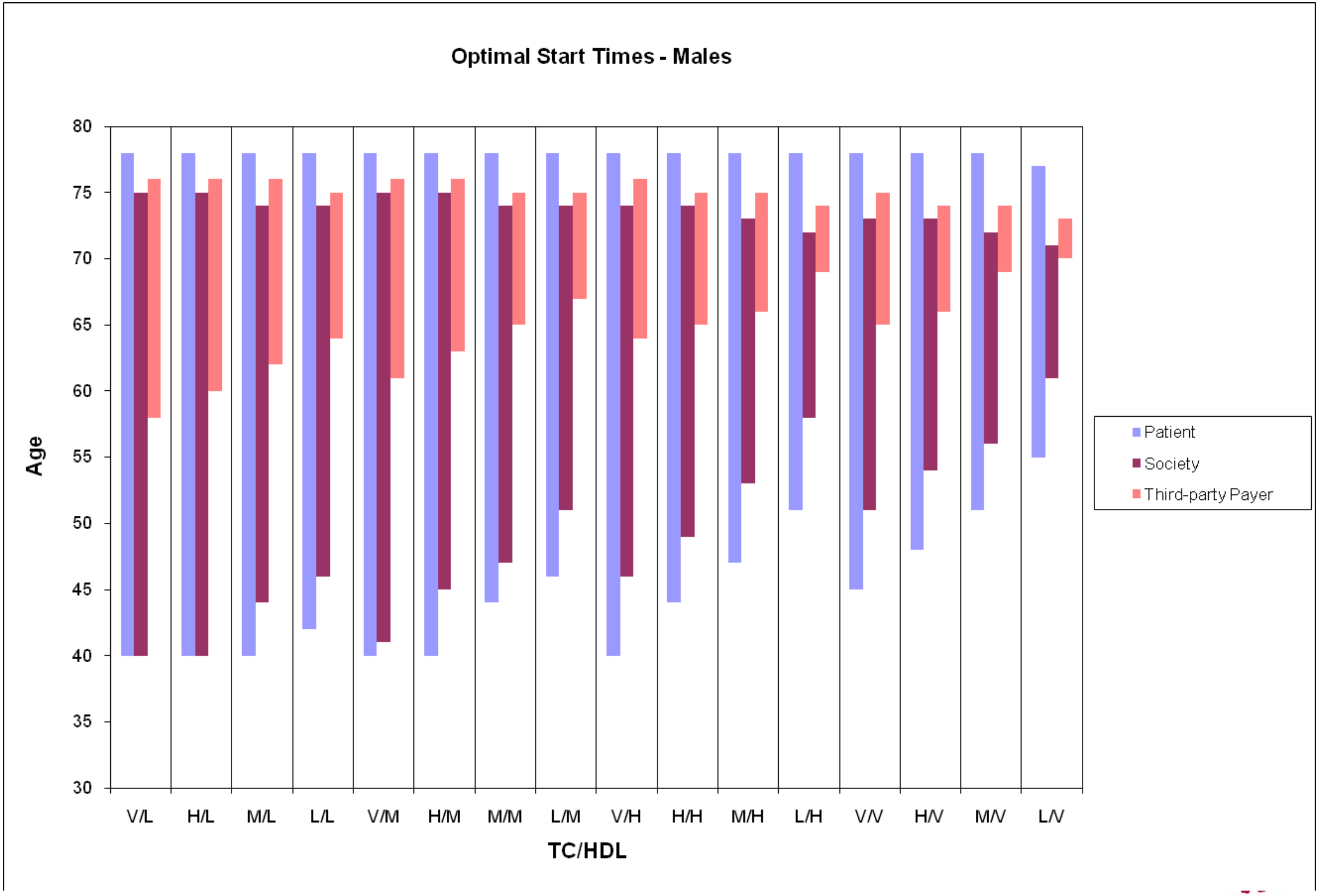
$$v_t(s_t, m) = \max \left[\underbrace{r(s_t, m)}_{\text{Reward for living to current epoch}} + \overbrace{\lambda \sum_{\forall s_{t+1}} \underbrace{p(s_t', m' \mid s_t, m)}_{\text{Transition probabilities}} v_t(s_t', m)}^{\text{Expected Future Reward}} \right]$$

Example

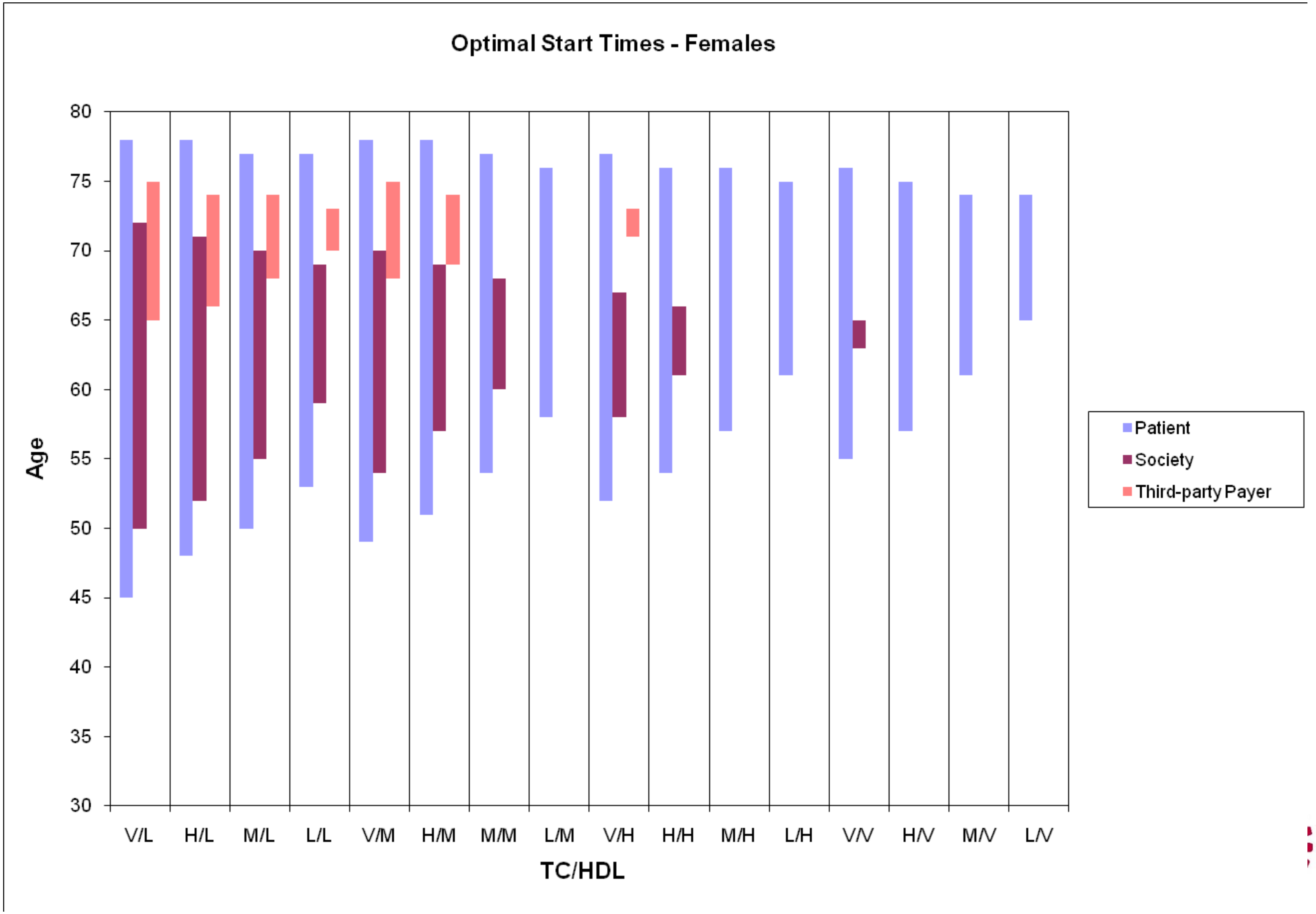
Optimal Start Times for Males with very high total cholesterol and low HDL.



Numerical Results



Numerical Results



Primary Prevention

- Define $\mu_t(s_t)$ to be the patients expected quality adjusted time to first event if treatment is initiated in state s_t
- States: defined by *lipid ratio*
- Objective: maximize time to first event ($r(s_t, m) = 1 - m\sigma$)
- Optimality equations:

$$v_t(s_t) = \max\left\{ r(s_t, 0) + \lambda \sum_{\forall s_{t+1}} p(s_{t+1}', 0 \mid s_t, 0) v_t(s_{t+1}'), \mu_t(s_t) \right\}$$

Theoretical Insights

Theorem 1: *If the transition probability matrix is IFR then the optimal policy has the control limit property such that for some lipid ratio* $z_t^* \in S'$:

$$a_t(s_t) = \begin{cases} 1 \text{ for } s_t \geq z_t^* \\ 0 \text{ for } s_t < z_t^* \end{cases}$$

Theorem 2: *If the expected benefit loss is nondecreasing in age then the optimal threshold is nonincreasing in age.*

Kurt, M., Denton, B.T., Schaefer, A., Shah, N., Smith, S., "At What Lipid Ratio Should a Patient with Type 2 Diabetes Initiate Statins", submitted to *Management Science*

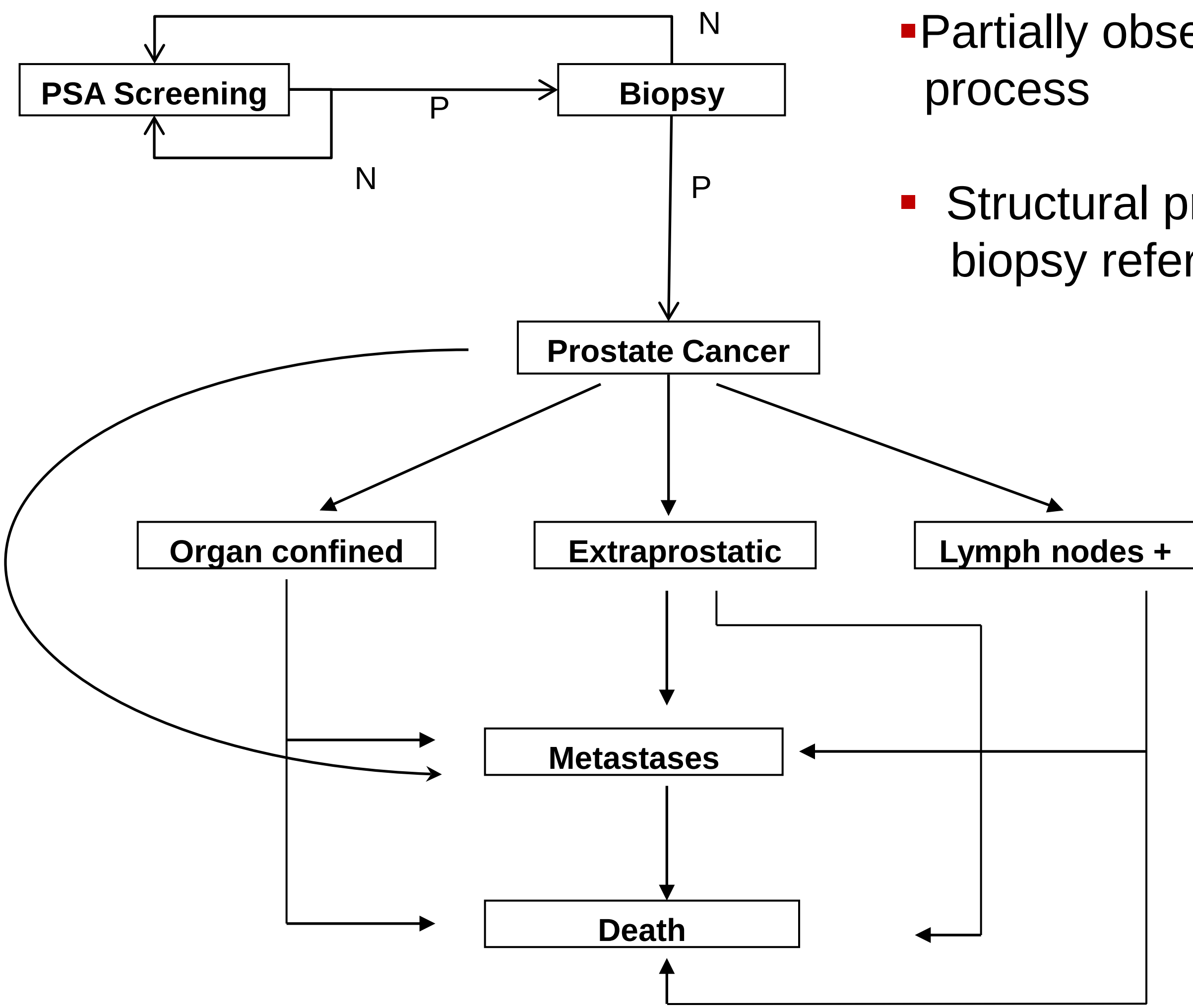
Other Research Directions

Treatment Optimization

- Optimal sequencing and timing of treatment options for control of:
  - Cholesterol
  - Blood pressure
  - HbA1c
- Barriers to treatment:
  - Adherence monitoring and control

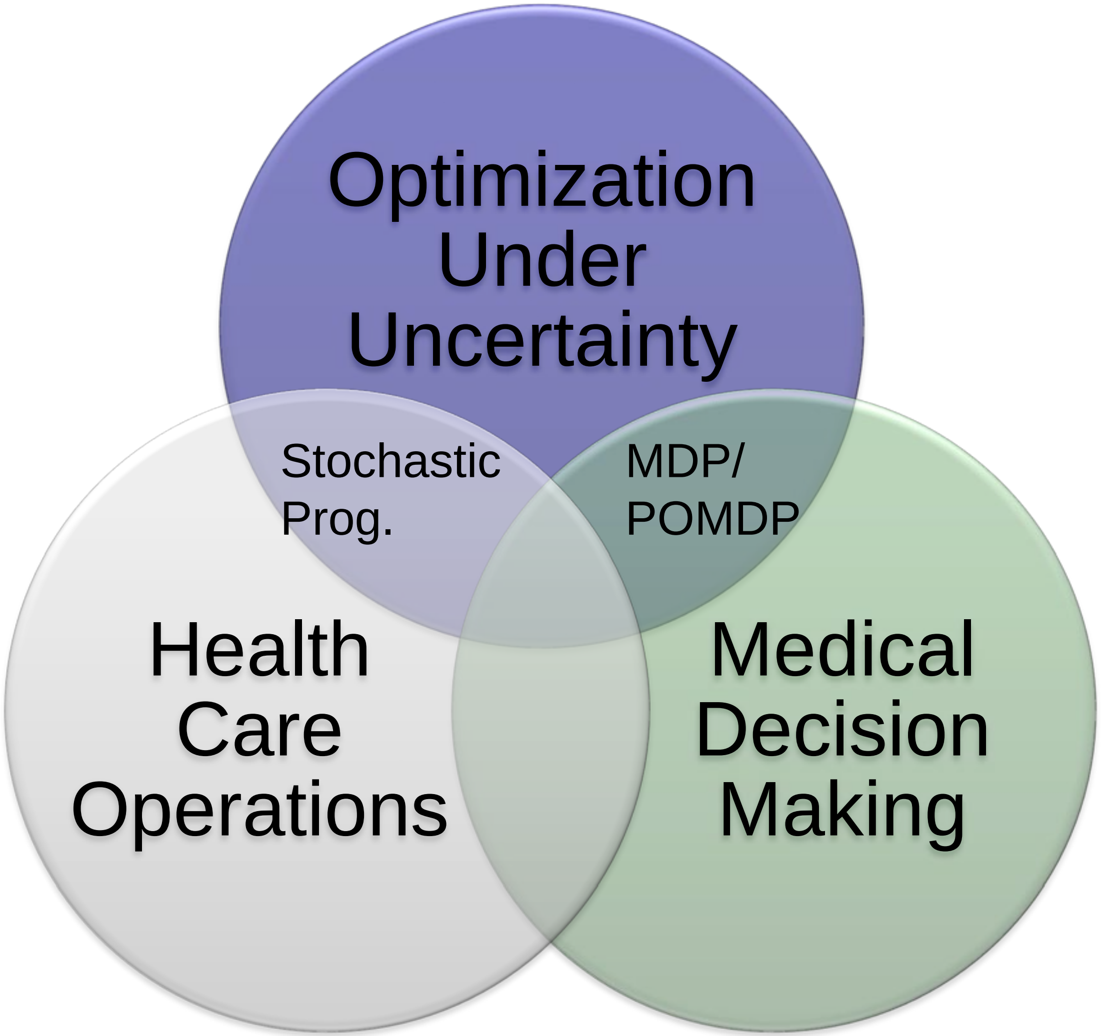
Mason, J.E., England, D., Denton, B.T., Smith, S.A., Shah, N.D., "The Effect of Adherence on the Optimal Start Time for Statins," Working Paper, 2009

Prostate Cancer Screening



- Partially observable Markov decision process
- Structural properties of the optimal biopsy referral policy
- Zhang, J., Denton, B.T., Balasubramanian, H., Inman, B., Shah, N., "Optimization of Prostate Biopsy Decisions," Working Paper, 2009

Future Opportunities



Questions?