Optimization of Health Care Delivery Systems

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Summary

- **Health Care Operations:**
  - Planning and scheduling of appointment-based service systems
  - Stochastic Programming and Robust Optimization

- **Medical Decision Making:**
  - Screening and treatment decisions for chronic diseases
  - Markov Decision Processes (MDPs) and Partially observable MDPs
Appointment-Based Service Systems
Collaborators

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Summary

- Optimization models for appointment based stochastic service systems
  - Single server scheduling
  - Multiple server scheduling
  - Dynamic scheduling
  - Bi-criteria scheduling
  - Scheduled service networks
Surgery

- **Patient Intake:** administrative activities, pre-surgery exam, gowning, site prep, anesthetic

- **Surgery:** incision, one or multiple procedures, pathology, closing

- **Recovery:** post-anesthesia care unit (PACU), ICU, hospital bed
Complicating Factors

- Multiple expensive resources
- Large number of activities to be completed in a fixed period of time
- Uncertainty in duration of activities
- Many competing criteria
Outpatient Procedure Center
Single Server Scheduling
Single OR Scheduling ($S(n)/G(n)/1$)

\[ \min \{ \text{Idling} + \text{Waiting} + \text{Overtime} \} \]
Stochastic Optimization Model

\[
\min \left\{ \sum_{i=1}^{n} c_i^w E_Z[w_i] + \sum_{i=1}^{n} c_i^s E_Z[s_i] + c^L E_Z[l] \right\}
\]

\[
\begin{align*}
  w_i &= \max(w_{i-1} + Z_{i-1} - x_{i-1}, 0), \quad i = 1, \ldots, n - 1 \\
  s_i &= \max(-w_{i-1} - Z_{i-1} + x_{i-1}, 0), \quad i = 1, \ldots, n - 1 \\
  l &= \max(w_n + Z_n + \sum_{i=1}^{n-1} x_i - d, 0)
\end{align*}
\]
Stochastic Linear Program

\[ \min \{E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\} \]

s.t. \[ w_2 - s_2 = Z_1 - x_1 \]
\[ - w_2 + w_3 - s_3 = Z_2 - x_2 \]
\[ - w_n - s_n + l - g = Z_n - d + \sum_{j=1}^{n-1} x_i \]
\[ x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1, \ldots, n, \quad l, g \geq 0 \]
Two Stage Recourse Problem

Initial Decision (x) $\rightarrow$ Uncertainty Resolved (Z) $\rightarrow$ Recourse (y)

$$\min \{Q(x) = E_Z[Q(x,Z)]\}$$

$$Q(x,Z^k) = \min \{c \cdot y^k \mid T x + W y^k = Z^k, y^k \geq 0\}$$
Example

- Schedule for $n=7$ with i.i.d. distributions with $U(1,2)$:

\[ X_i \]

\[ \mu = 1.5 \]


Multi-OR Surgery Allocation
Outpatient Procedure Center
Multi-OR Room Scheduling

Performance Measures:
- Cost of opening ORs
- Overtime costs

Decisions:
- How many ORs to open?
- Which OR to schedule each surgery block?
Extensible Bin Packing

\[ x_j = \begin{cases} 
1 & \text{if OR } j \text{ open} \\
0 & \text{if OR } j \text{ closed} 
\end{cases} \]

\[ y_{ij} = \begin{cases} 
1 & \text{if Surgery } i \text{ assigned to OR } j \\
0 & \text{Otherwise} 
\end{cases} \]

\[ Z = \min \{ \sum_{j=1}^{m} c^f x_j + c^v o_j \} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} b_i y_{ij} - o_j \leq d_j x_j \quad \forall (i, j) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j \geq 0 \]
Two-Stage Stochastic MIP

\[ Q(\mathbf{x}) = \min \left\{ \sum_{j=1}^{m} c^f x_j + c^v E_\omega [o_j(\omega)] \right\} \]

s.t. \[ y_{ij} \leq x_j \quad \forall (i, j) \]

\[ \sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i) \]

\[ \sum_{i=1}^{n} b_i(\omega) y_{ij} - o_j(\omega) \leq d_j x_j \quad \forall (i, j, \omega) \]

\[ y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \geq 0, \forall \omega \]
General Insights

- New upper and lower bounds on the extensible bin-packing problem
- Valid inequalities to reduce the impact of symmetry
- Decomposition based solution methods
- New Robust MIP formulation

Other Research Directions
Dynamic Appointment Scheduling

- Patients request appointments stochastically
- Appointment decisions are made one at a time
- Multistage stochastic linear program

POD Scheduling

Chronic Disease Screening and Treatment
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Summary

- Disease screening under imperfect information
  - Biopsy referral decisions
  - National screening policies

- Optimal timing of medical treatment decisions
  - Treatment optimization
  - Adherence control
Disease Test Beds

- **Type 2 Diabetes:**
  - Mayo Clinic DEMS
  - Ingenix Inc.

- **Prostate Cancer:**
  - Olmsted county medical record
  - Mayo Clinic radical prostatectomy database

- **Bladder cancer**
  - UNC Medical Record
  - SEER
Diabetes

- There are more than 23 million people in the U.S. who have diabetes
  - 8% of the U.S. population
  - 90% have type 2 diabetes

- Two out of three people with diabetes will die from either stroke or coronary heart disease (CHD)
Cost of Statins

- More than $20 billion dollars are spent annually in the U.S. alone
- Side effects including liver failure, muscle pain, dizziness, nausea, headaches...
- Currently there is broad disagreement about the best policy for statin treatment
CHD & Stroke Risks

• What other factors affect CHD and Stroke risk?
  • Age
  • Gender
  • Ethnicity
  • Smoking
  • Blood Pressure
  • Hemoglobin A1c
  • Exercise & Diet
  • Body Mass Index

• In recent years several risk models have been developed to calculate the risk of CHD & Stroke:
  • UKPDS
  • Framingham
  • Archimedes
Markov Decision Process Model

- **Stages:**
  - Time horizon: Ages 40-100
  - Annual decision epochs

- **Decision:**
  - Initiate or delay statin treatment

- **States:**
  - Metabolic: Total cholesterol and HDL (each can be L, M, H, V), Blood pressure, HbA1c
  - Demographic: Gender, Race, BMI, smoking status, medical history
Non-Fatal Events (On Statins)

On Statins

Metabolic States before an event has occurred.

Death
Optimal Treatment Policy

- **Society**
  - Maximize a weighted combination of patient rewards for life years minus costs of treatment and health services

- **Patient**
  - Maximize rewards for quality adjusted life years

- **Third-party Payer**
  - Minimize costs of treatment and health services

Model Formulation

- Decision epochs: \( t \in T = \{1, 2, 3, \ldots, N\} \)
- Health States: \( s_t \in S = \{1, 2, 3, \ldots, L, L + 1\} \)
- Treatment Status: \( m \in \{0, 1\} \)

- Action: \( a_t(s_t) = \begin{cases} I, W & \text{if } m = 0 \\ W & \text{if } m = 1 \end{cases} \)

Optimality Equations:

\[
v_t(s_t, m) = \max \left[ r(s_t, m) + \lambda \sum_{s_{t+1}} p(s_t', m' | s_t, m) v_t(s_t', m) \right]
\]

Expected Future Reward

Reward for living to current epoch

Transition probabilities
Example

Optimal Start Times for Males with very high total cholesterol and low HDL.
Numerical Results

Optimal Start Times - Males

TC/HDL

Age

Patient
Society
Third-party Payer
Numerical Results

Optimal Start Times - Females

Age

TC/HDL

- Patient
- Society
- Third-party Payer
Primary Prevention

- Define $\mu_t(s_t)$ to be the patients expected quality adjusted time to first event if treatment is initiated in state $S_t$
- States: defined by lipid ratio
- Objective: maximize time to first event $\left( r(s_t, m) = 1 - m\sigma \right)$
- Optimality equations:

$$v_t(s_t) = \max \left\{ r(s_t, 0) + \lambda \sum_{s_{t+1}} p(s_{t+1}', 0 \mid s_t, 0) v_t(s_{t+1}'), \mu_t(s_t) \right\}$$
Theoretical Insights

**Theorem 1:** If the transition probability matrix is IFR then the optimal policy has the control limit property such that for some lipid ratio $z^*_t \in S'$:

$$
\alpha_t(s_t) = \begin{cases} 
1 & \text{for } s_t \geq z^*_t \\
0 & \text{for } s_t < z^*_t 
\end{cases}
$$

**Theorem 2:** If the expected benefit loss is nondecreasing in age then the optimal threshold is nonincreasing in age.

Other Research Directions
Treatment Optimization

- Optimal sequencing and timing of treatment options for control of:
  - Cholesterol
  - Blood pressure
  - HbA1c

- Barriers to treatment:
  - Adherence monitoring and control

Prostate Cancer Screening

- Partially observable Markov decision process
- Structural properties of the optimal biopsy referral policy

Future Opportunities

Optimization Under Uncertainty

- Stochastic Prog.
- MDP/POMDP

Health Care Operations

Medical Decision Making
Questions?