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Optimization of Scheduling for Appointment Based Healthcare Delivery Systems

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December 2, 2011

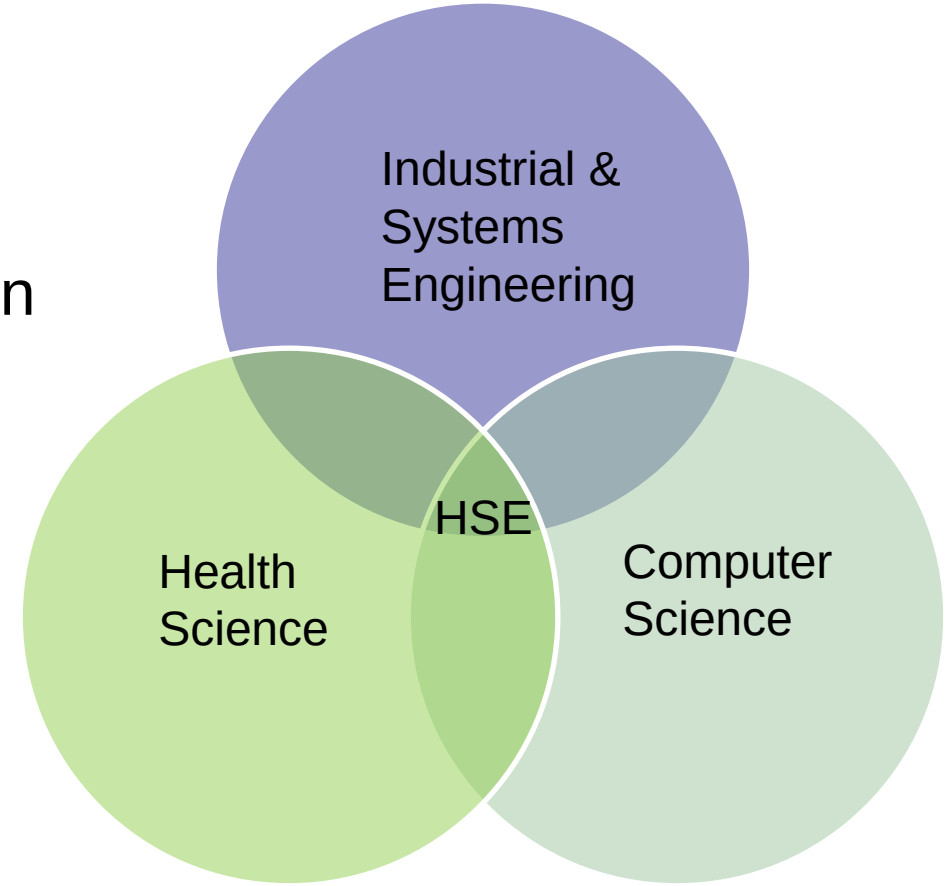
Health Systems Engineering

Simulation

Optimization

Decision Analysis

Quality Control



Medicine and Health Policy

Operations Management

Hospital Logistics

Decision Support Systems

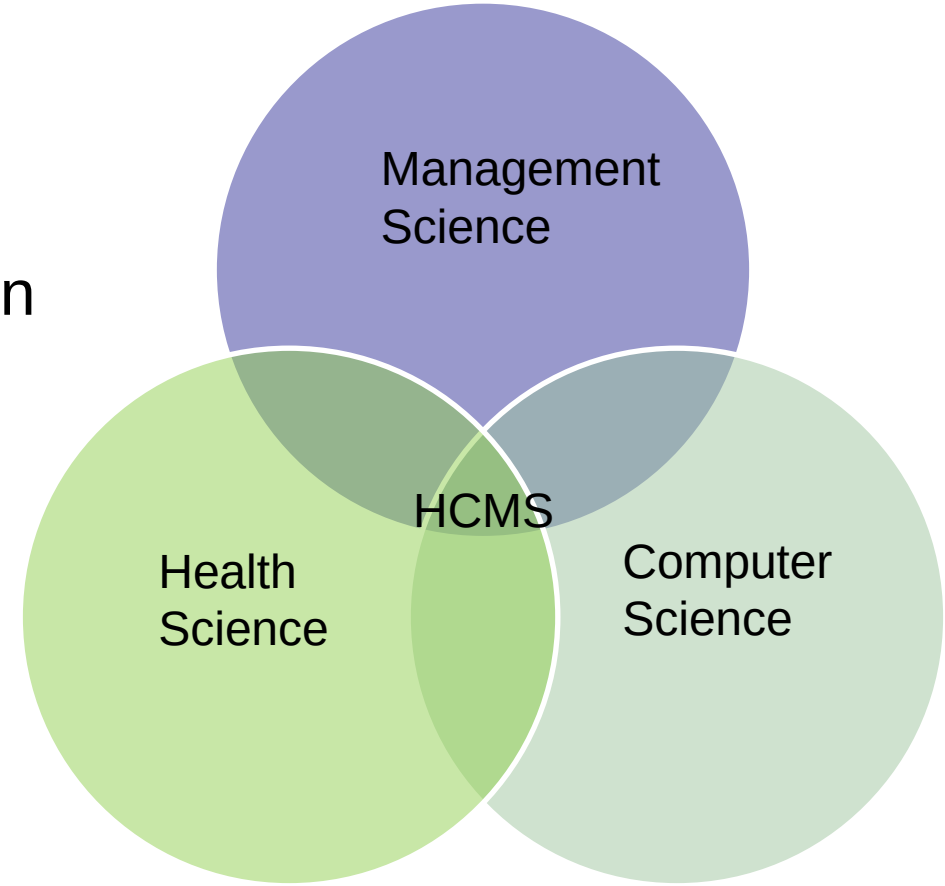
Health Care Management Science

Simulation

Optimization

Decision Analysis

Quality Control



Medicine and Health Policy

Operations Management

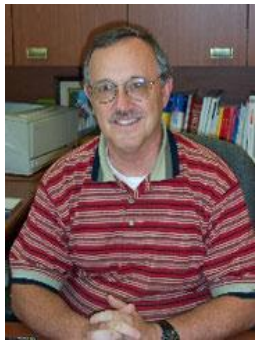
Hospital Logistics

Decision Support Systems

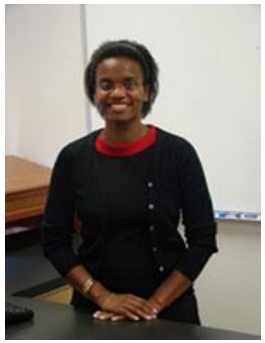
NCSU Industrial & Systems Engineering

- 4th Largest Industrial & Systems Engineering department in the U.S.
- Ranked 13th by U.S. News & World Report
- Undergraduate, Masters and PhD programs
- In 2006, a self-study identified Healthcare as a “focus area”

Health Systems Faculty



Steve Roberts



Julie Ivy

Javad Taheri

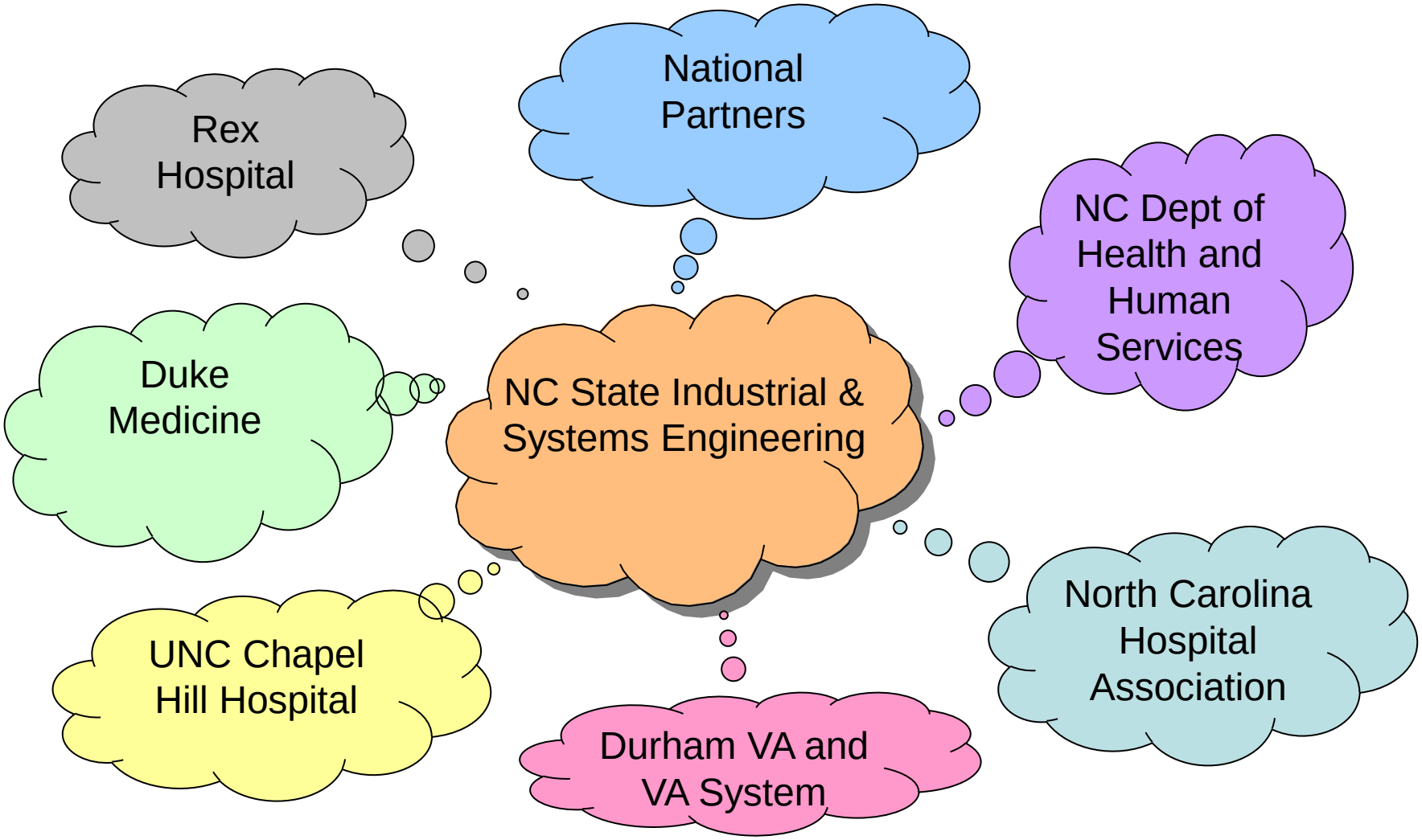
Brian Denton

+ Many other faculty specializing in biomanufacturing, human factors engineering, and other areas

Application Interests

Methodological Interests

Collaborators



Summary

- Surgery process and complicating factors
- 3 Problems:
  - Single OR scheduling
  - Multi OR planning and surgery allocation
  - Scheduling of an Outpatient Procedure Center
- Future research

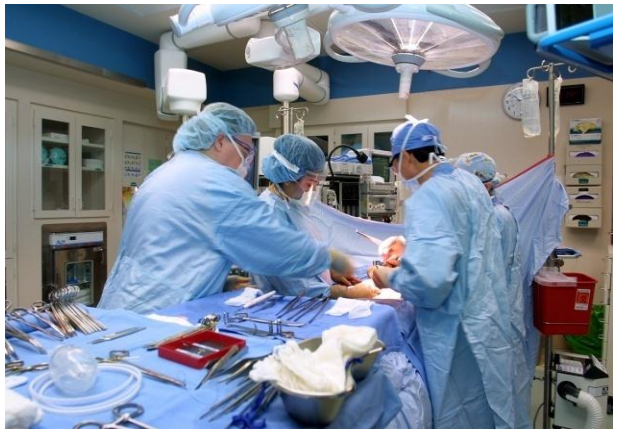
Collaborators

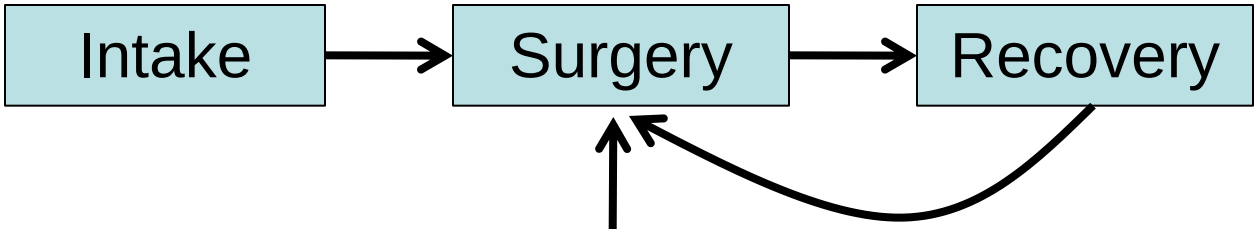
Hari Balasubramanian (University of Massachusetts)
Sakine Batun (University of Pittsburgh)
Bjorn Berg (NCSU)
Ayca Erdogan (NCSU)
Todd Huschka (Mayo)
Andrew Miller (University of Bordeaux)
Heidi Nelson (Mayo)
Ahmed Rahman (Mayo)
Andrew Schaefer (University of Pittsburgh)

Supported by National Science Foundation – CMMI 0620573

Surgery Process

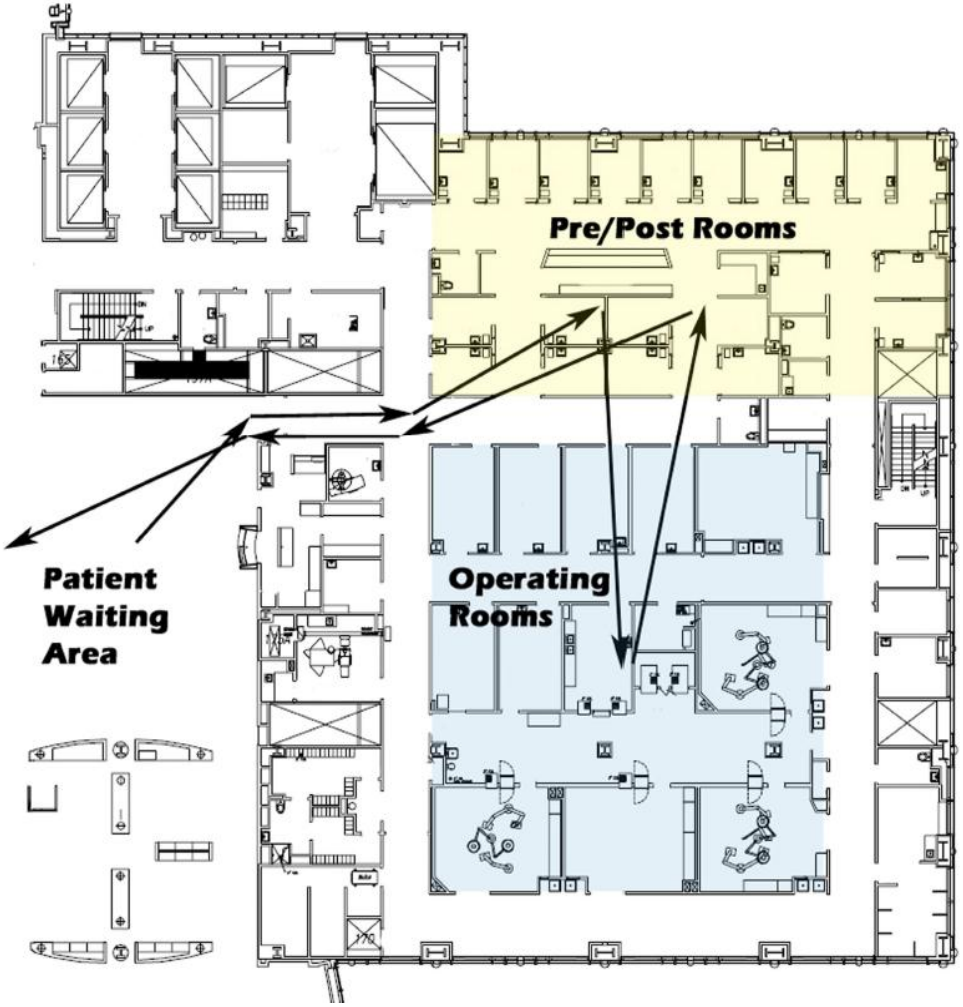
- Patient Intake: administrative activities, pre-surgery exam, gowning, site prep, anesthetic
- Surgery: incision, one or multiple procedures, pathology, closing
- Recovery: post anesthesia care unit (PACU), ICU, hospital bed





Outpatient Procedure Centers

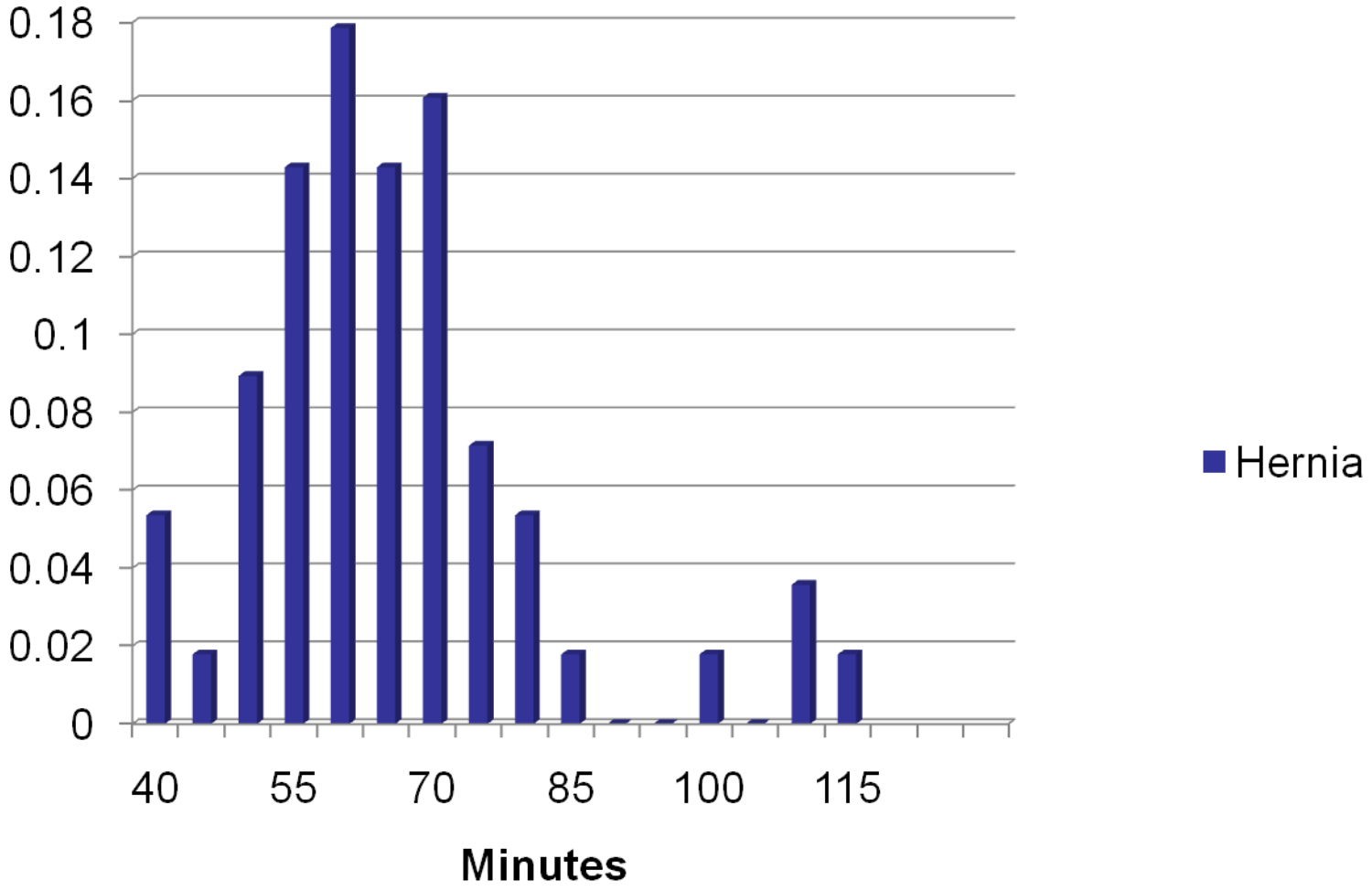
- Daily OR and staff planning
- Surgery-to-OR assignments
- Staff scheduling
- Patient arrival time scheduling
- Planning equipment resources (surgical kits, diagnostic equipment)



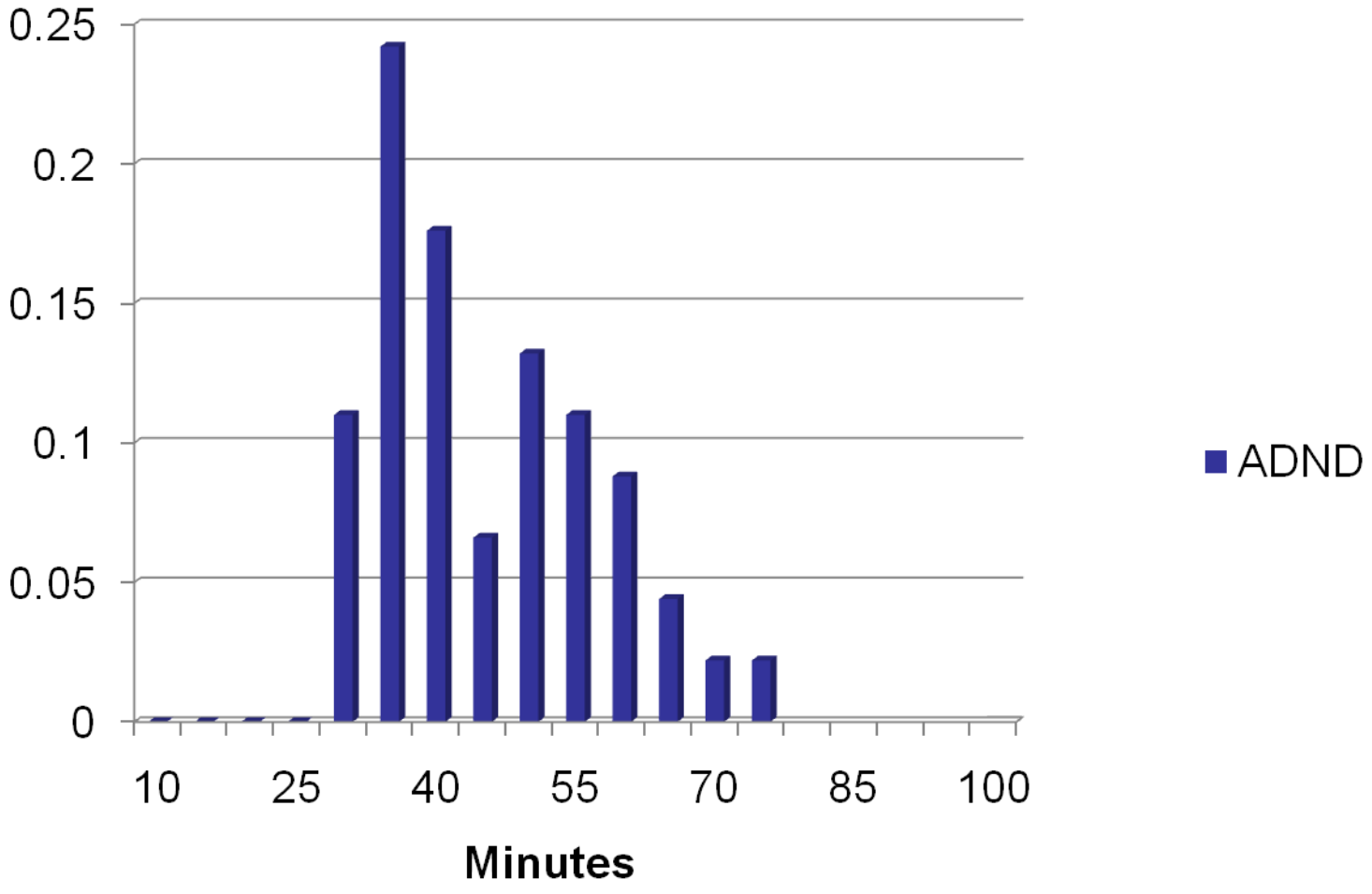
Complicating Factors

- Many types of resources to be scheduled: OR team, patients, equipment, materials
- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Many competing criteria

Surgery Duration Uncertainty

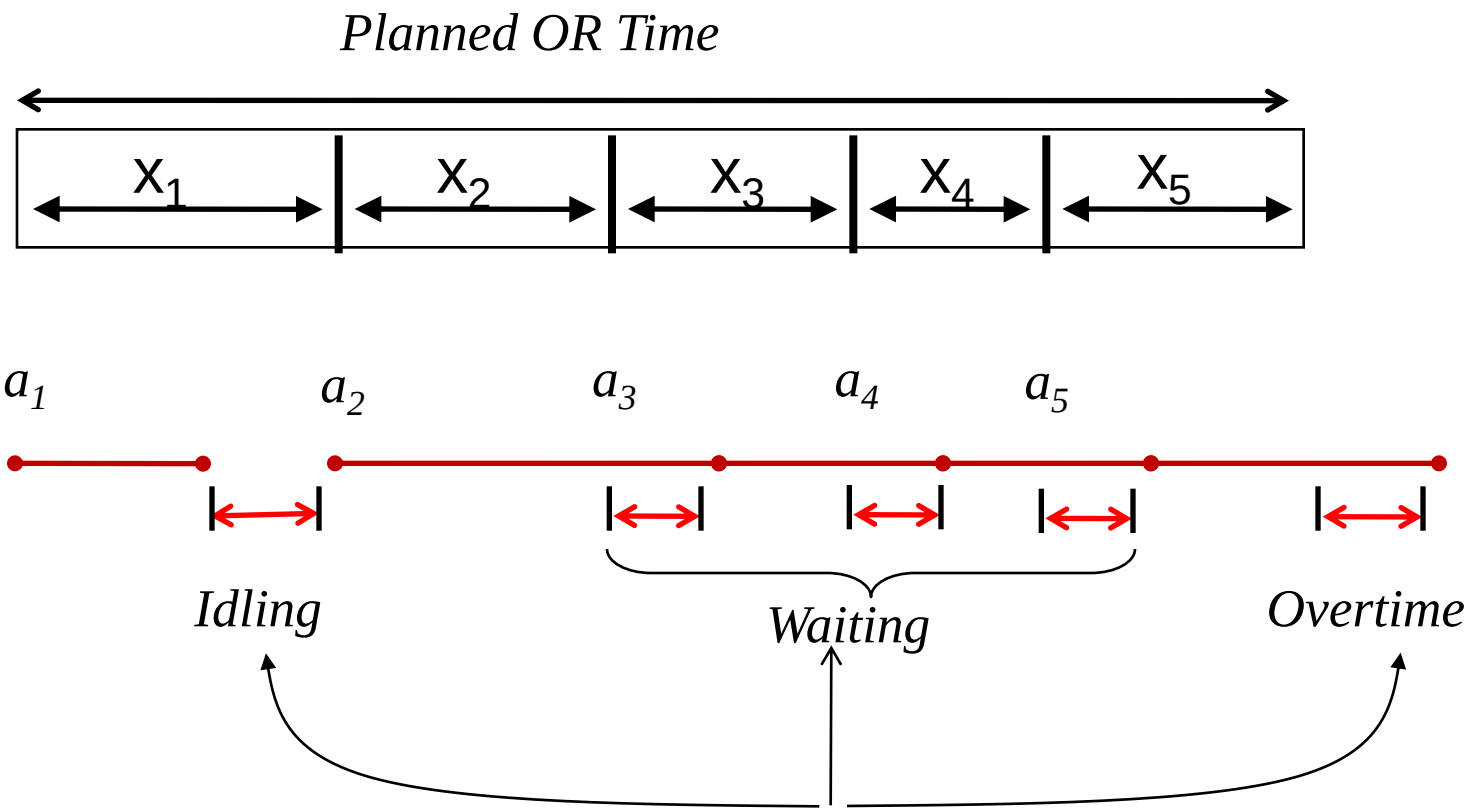


Surgery Duration Uncertainty



Problem 1: Single OR Scheduling

Single OR Scheduling - S(n)/G(n)/1



Min{ Idling + Waiting + Overtime}

Stochastic Optimization Model

$$\min\{\underbrace{\sum_{i=1}^{n} C_i^w * E_{\mathbf{Z}}[W_i]}_{\text{Cost of Waiting}} + \underbrace{\sum_{i=1}^{n} C^s * E_{\mathbf{Z}}[S_i]}_{\text{Cost of Idling}} + \underbrace{C^L * E_{\mathbf{Z}}[L]}_{\text{Cost of Overtime}}\}$$

$$W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)$$

$$S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)$$

$$L = \max(W_n + Z_n + \Sigma x_i - d, 0)$$

Literature Review – Single Server

- Queuing Analysis:
  - Mercer (1960, 1973)
  - Jansson (1966)
  - Brahimi and Worthington (1991)
- Heuristics:
  - White and Pike (1964)
  - Soriano (1966)
  - Ho and Lau (1992)
- Optimization:
  - Weiss (1990) – 2 surgery news vendor model
  - Wang (1993) – Exploited phase type distribution property
  - Denton and Gupta (2003) – General 2-SLP formulation

Stochastic Linear Program

$$\min\{ E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\}$$

s.t.

$$\begin{aligned}
w_2 \quad - s_2 \quad &= Z_1 - x_1 \\
-w_2 + w_3 \quad - s_3 \quad &= Z_2 - x_2 \\
\ddots \qquad \ddots \quad & \\
-w_n \quad - s_n + l - g &= Z_n - d + \sum_{j=1}^{n-1} x_i
\end{aligned}$$

$$x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1,\ldots,n, \quad l, g \geq 0$$

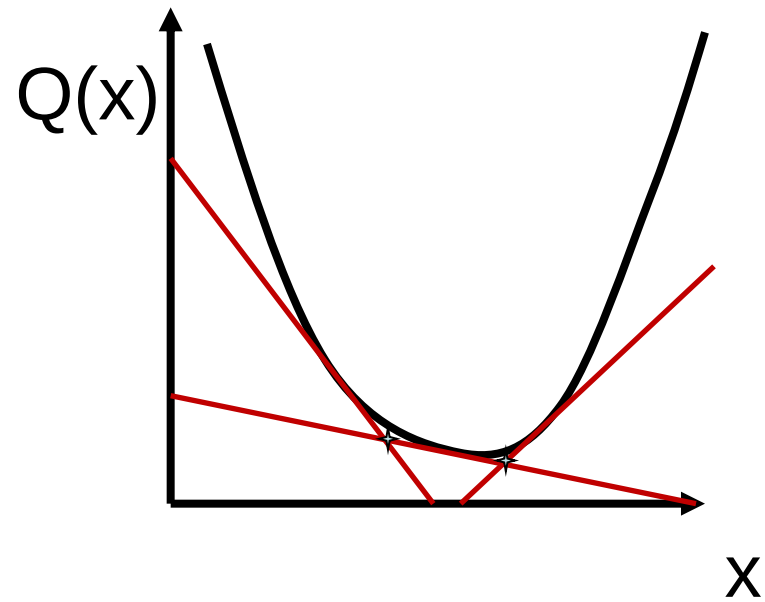
Two Stage Recourse Problem

Initial Decision (x) → Uncertainty Resolved → Recourse (y)

$$\min\{\, Q(\mathbf{x}) = E_{\mathbf{Z}}[Q(\mathbf{x},\mathbf{Z})]\}$$

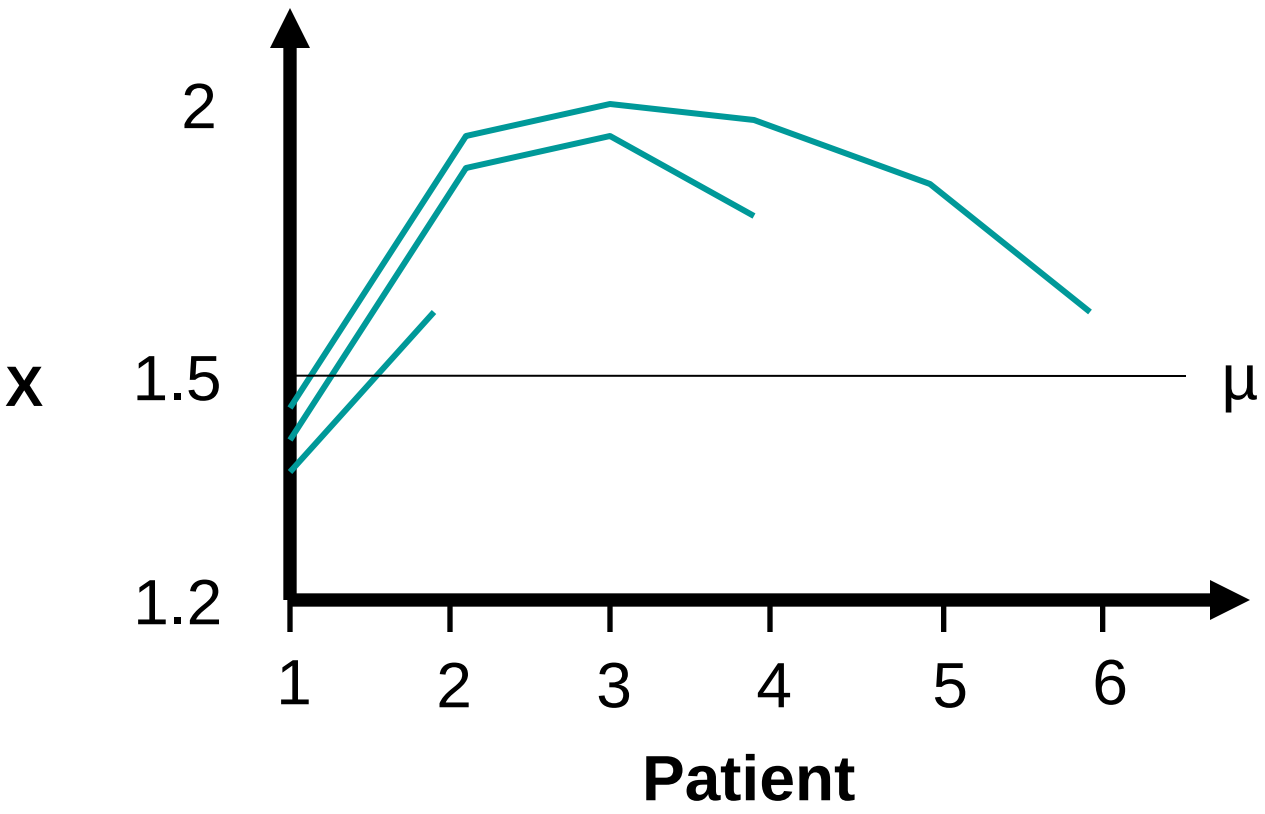
$$Q(\mathbf{x},\mathbf{Z}^k) = \min\{\, \mathbf{c}\cdot\mathbf{y}^k \,|\, \underbrace{T\,\mathbf{x} + W\,\mathbf{y}^k}\, = \mathbf{h}^k, \mathbf{y}^k \geq 0\}$$

$$\begin{pmatrix} T & W^1 & & & \\ T & & W^2 & & \\ T & & & W^3 & \\ \vdots & & & & \ddots \\ T & & & & & W^K \end{pmatrix}$$



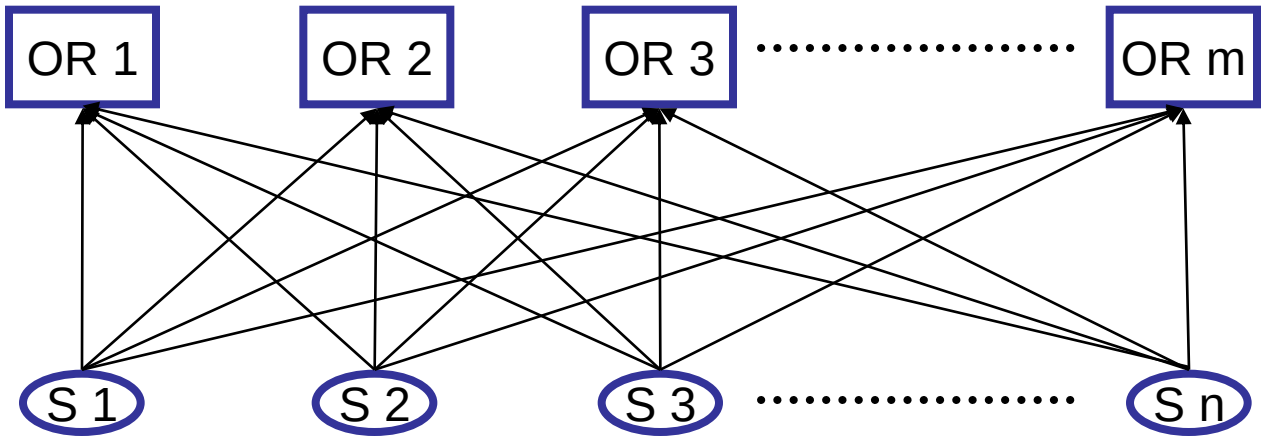
Example

- Comparison of surgery allocations for n=3, 5, 7 with i.i.d. distributions with U(1,2):



Problem 2: Multi-OR Surgery Allocation

Multi-Operating Room Scheduling



Decisions:

- How many operating rooms (ORs) to open?
- Which OR to schedule each surgery block in?

Performance Measures:

- Cost of operating rooms opened
- Overtime costs for operating rooms

Extensible Bin Packing

$$x_j = \begin{cases} 1 & if\ OR\ \ j\ open \\ 0 & if\ \ OR\ \ j\ closed \end{cases} \qquad y_{ij} = \begin{cases} 1 & if\ Surg.\ Block\ \ i\ assigned\ \ to\ OR\ \ j \\ 0 & Otherwise \end{cases}$$

$$Z = \min\{ \sum_{j=1}^{m} c^f x_j + c^v o_j \}$$

$$s.t. \qquad y_{ij} \le x_j \quad \forall (i, j)$$

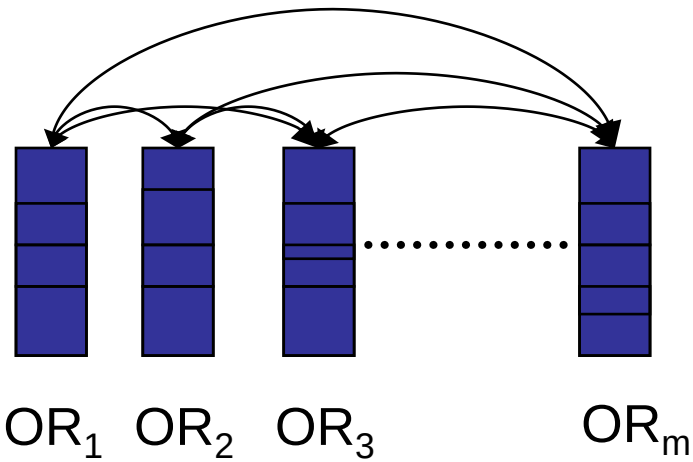
$$\sum_{j=1}^{m} y_{ij} = 1 \ \ \forall (i)$$

$$\sum_{i=1}^{n} \bar{z}_i y_{ij} - o_j \le d_j x_j \quad \forall (i, j)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j \ge 0$$

Symmetry

- m! optimal solutions:



- Anti-symmetry constraints:

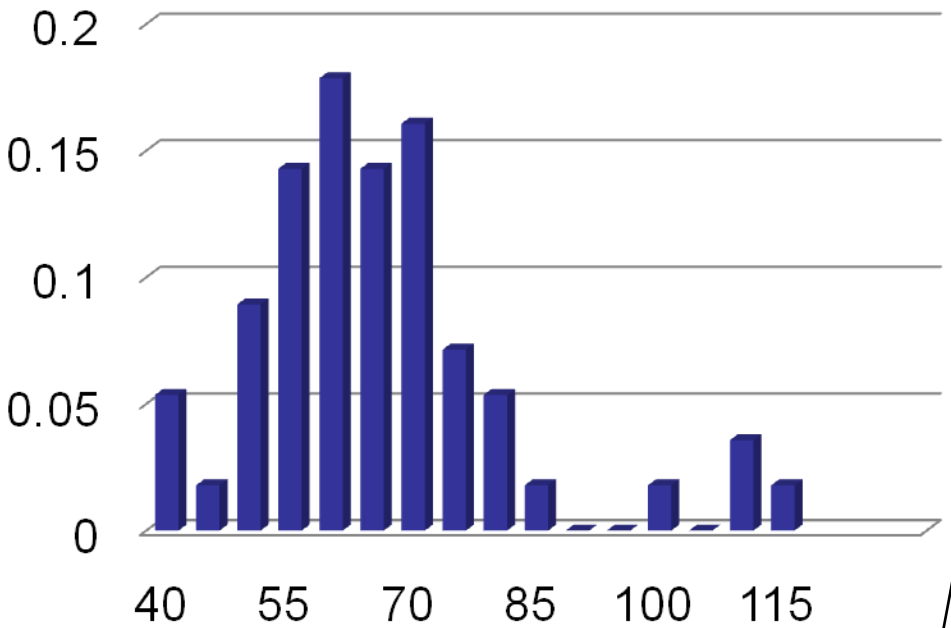
$$x_1 \geq x_2$$
$$x_2 \geq x_3$$
$$\vdots$$
$$x_m \geq x_{m-1}$$

OR Ordering

$$y_{11} = 1$$
$$y_{21} + y_{22} = 1$$
$$\vdots$$
$$\sum_{j=1}^{m} y_{mj} = 1$$

Surgery Assignment

Two-Stage Stochastic MIP



$$Q(\mathbf{x}) = \min\{ \sum_{j=1}^{m} c^f x_j + c^v E_\omega[o_j(\omega)]\}$$

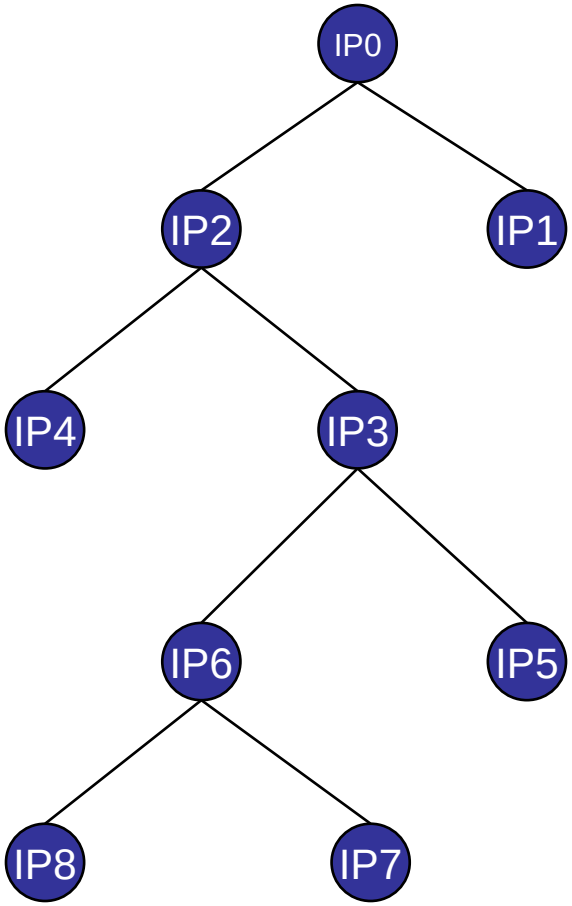
$$s.t. \quad y_{ij} \le x_j \quad \forall (i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$\sum_{i=1}^{n} z_i(\omega) y_{ij} - o_j(\omega) \le d x_j \quad \forall (i,j,\omega)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \ge 0, \forall \omega$$

Integer L-Shaped Method



Master Problem:

$$Z = \min\{ \sum_{j=1}^{m} c^f x_j + \Theta \}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_j \in \{0,1\}, \Theta \ge 0$$

$$\Theta \ge E_{\omega}[\pi(h - Tx)]$$

Heuristic

Longest processing time first heuristic:

EBP Heuristic:

$$n \leftarrow LB;$$

$$repeat;$$

$$LPT(n);$$

$$if\ (o_j = 0, \forall j)\ Stop;$$

$$n \leftarrow n+1;$$

$$end(repeat);$$

$$LB = \left\lceil \frac{\sum_{i=1}^{n} \bar{z}_i}{T(1+\frac{c^f}{c^v T})} \right\rceil$$

- Sort surgeries from longest to shortest
- Sequentially apply surgeries to emptiest room

Robust Formulation

$$Z = \min\{ \sum_{j=1}^{m} c^f x_j + F(x, y)\}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_j \in \{0,1\} \ge 0$$

$$F(x, y) = \begin{cases} \max_{\delta} \{ \sum_{j=1}^{m} \eta_j \} \\ s.t. \quad \eta_j = c_j^v \max\{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - T_j x_j\}, \quad \forall j \\ \sum_{(i,j):y_{ij}=1}^{m} \frac{\delta_{ij} - \underline{z}_i}{\bar{z}_i - \underline{z}_i} y_{ij} \le \tau \\ \underline{z}_i \le \delta_{ij} \le \bar{z}_i, \forall (i, j) : y_{ij} = 1 \end{cases}$$

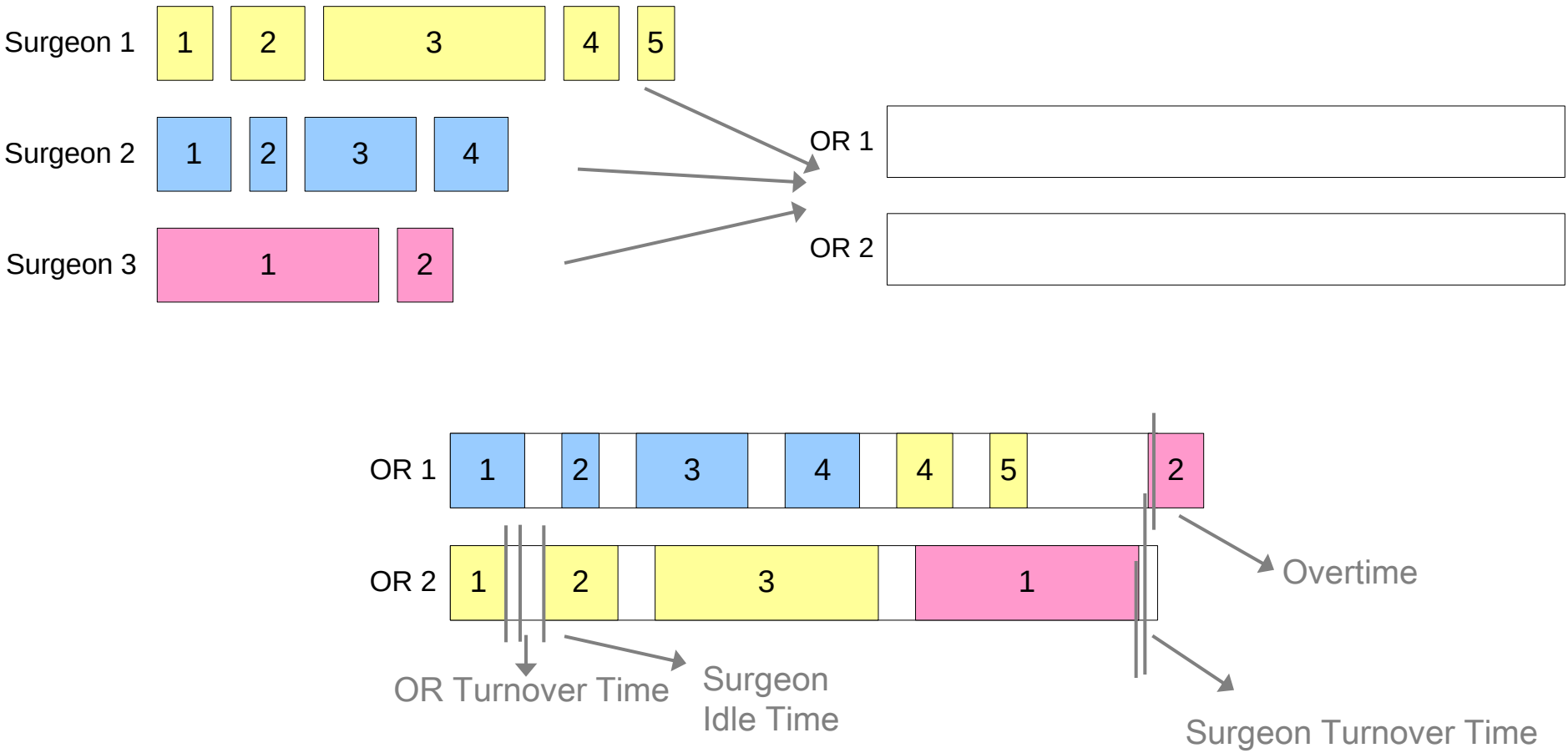
	15 surgery instances									
	Variable Cost = 0.033					Variable Cost = 0.0083				
			Robust IP					Robust IP		
Instance	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6	MV_IP	LPT_Heu	Tau=2	Tau=4	Tau=6
1	0.808	0.806	0.892	0.906	0.933	0.999	0.998	0.880	0.948	0.948
2	0.953	0.966	0.898	0.896	0.970	0.999	0.999	0.999	0.999	0.980
3	0.854	0.852	0.936	0.937	0.970	0.999	0.999	0.929	0.952	0.944
4	0.925	0.972	0.911	0.971	0.917	0.999	0.998	0.930	0.930	0.929
5	0.896	0.946	0.831	0.916	0.892	0.990	0.996	0.932	0.938	0.924
6	0.862	0.853	0.923	0.931	0.938	0.989	0.990	0.886	0.881	0.881
7	0.930	0.936	0.810	0.930	0.817	0.973	0.993	0.844	0.974	0.927
8	0.888	0.966	0.876	0.903	0.904	0.966	0.966	0.966	0.987	0.939
9	0.962	0.966	0.964	0.969	0.964	0.975	0.993	0.847	0.960	0.957
10	0.860	0.924	0.910	0.893	0.918	0.997	0.996	0.900	0.901	0.903
average	0.894	0.919	0.895	0.925	0.922	0.988	0.993	0.916	0.951	0.933
stdev	0.046	0.057	0.047	0.028	0.046	0.013	0.010	0.059	0.045	0.028
max	0.962	0.972	0.964	0.971	0.970	0.999	0.999	1.042	1.040	0.980

General Insights

- The LPT based heuristic works (fairly) well on a large number of instances
  - LPT works very well when overtime costs are low
  - LPT is better (and easier) than solving MV problem in most cases
- Robust IP is better than LPT when overtime costs are high

Denton, B.T., Miller, A., Balasubramanian, H., Huschka, T. “Optimal Allocation of Surgery Blocks to Operating Rooms Under Uncertainty,” *Operations Research*, 58(4), 802-816, 2010

Current Research: Sharing ORs



Batun, S., Denton, B.T., Huschka, T.R., Schaefer, A.J., “The Benefit of Pooling Operating Rooms Under Uncertainty,” *INFORMS Journal on Computing* , 23(2), 220-237, 2011.

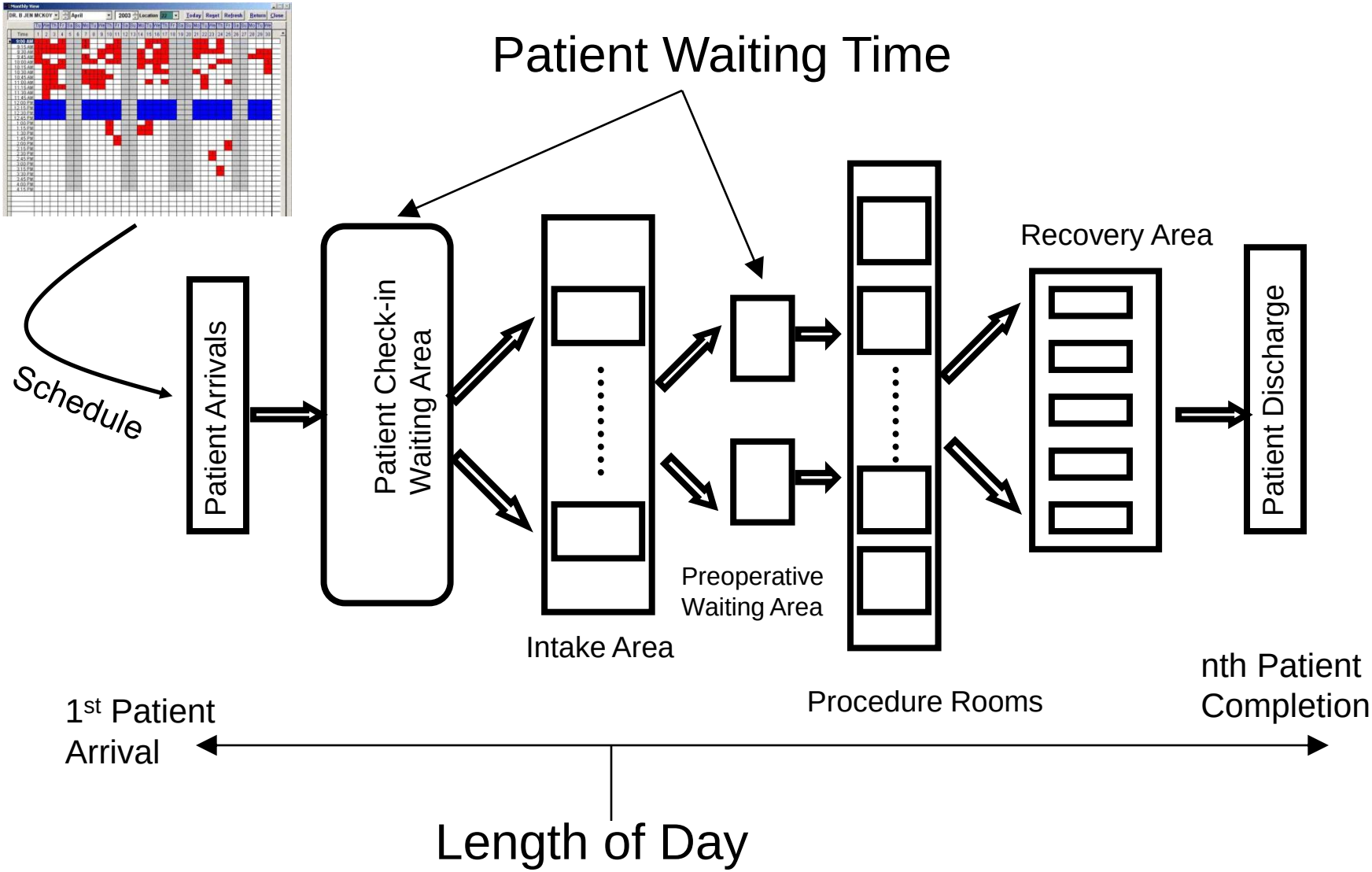
Problem 3: Patient Arrival Scheduling

Background

- Endoscopy practice at Mayo Clinic
- 4 endoscopists sharing 8 procedure rooms
- Approximately 45 patients per day
- Day begins at 7:30 AM and finishes at 5:00 PM

Berg, B., Denton, B.T., Nelson, H., Balasubramanian, H., Rahman, ., Bailey, A., Lindor, K., “A Computer Simulation Model to Evaluate Operational Performance of a Colonoscopy Suite,” *Medical Decision Making ,* 30(3), 380-387, 2010

Endoscopy Suite



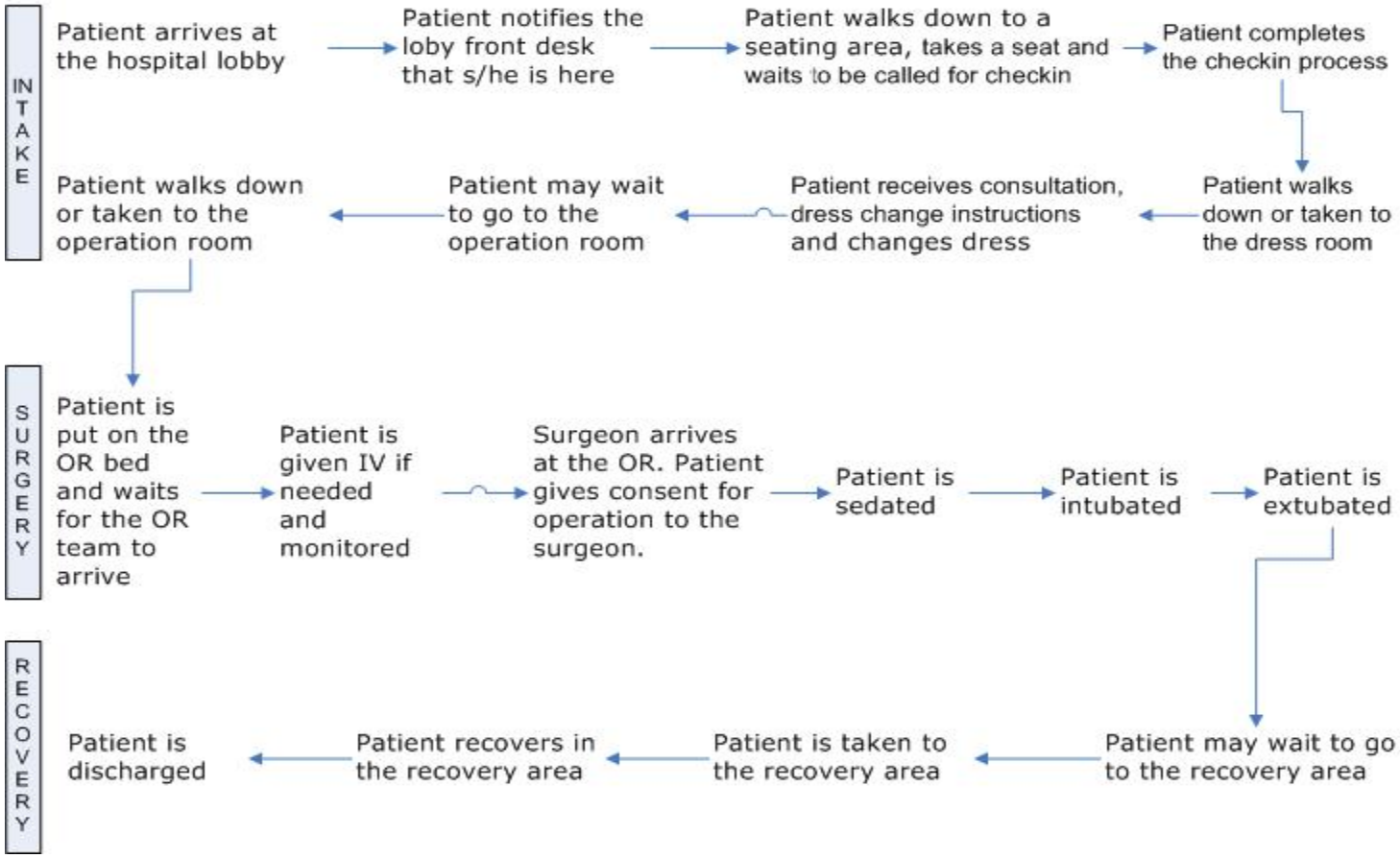
Performance Measures

- Patient throughput
- Waiting time:
  - Patients
  - OR Team
- Utilization:
  - Procedure room
  - OR Team
  - Recovery beds
- Overtime

Model Building

- Process Map:
  - Sequence of activities
  - Patterns of resource utilization
  - Decision points
- Conceptual Model
- Quantitative Models

Process Map


INTAKE
Patient arrives at the hospital lobby
Patient notifies the loby front desk that s/he is here
Patient walks down to a seating area, takes a seat and waits to be called for checkin
Patient completes the checkin process
Patient walks down or taken to the dress room
Patient receives consultation, dress change instructions and changes dress
Patient may wait to go to the operation room
Patient walks down or taken to the operation room
SURGERY
Patient is put on the OR bed and waits for the OR team to arrive
Patient is given IV if needed and monitored
Surgeon arrives at the OR. Patient gives consent for operation to the surgeon.
Patient is sedated
Patient is intubated
Patient is extubated
RECOVERY
Patient may wait to go to the recovery area
Patient is taken to the recovery area
Patient recovers in the recovery area
Patient is discharged

Data

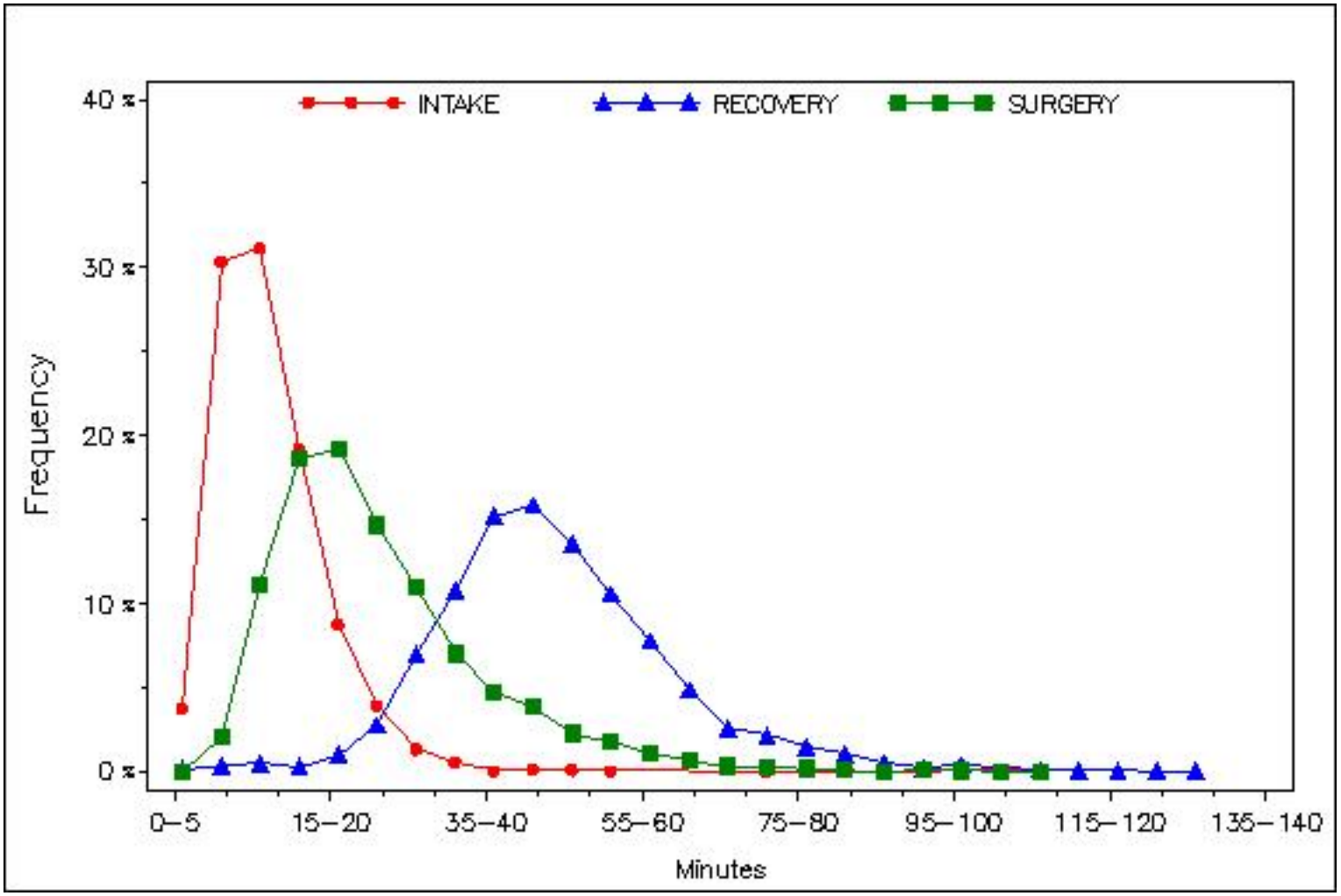
Instance_ID	Location	Room	Proc	Endoscopist	Primary Nurse	Appt Time	Pt. Status	Status time.
00001						12/6/05 7:30	WAITING	7:27
00001						12/6/05 7:30	CHECK_IN	7:42
00001						12/6/05 7:30	HOLDING	7:47
00001	Building	250	Colon	Dr.X	Nurse Y	12/6/05 7:30	ROOMING	7:47
00001	Building	250	Colon	Dr.X	Nurse Y	12/6/05 7:30	MD_IN_ROOM	8:04
00001	Building	250	Colon	Dr.X	Nurse Y	12/6/05 7:30	INTUBATION	8:10
00001	Building	250	Colon	Dr.X	Nurse Y	12/6/05 7:30	EXTUBATION	8:23
00001						12/6/05 7:30	BEDDED	8:28
00001						12/6/05 7:30	DISCHARGED	9:04
00001						12/6/05 7:30	SIGN_OFF	12:22

Intake

Procedure

Recovery

Intake, Surgery, and Recovery



Simulation-Optimization

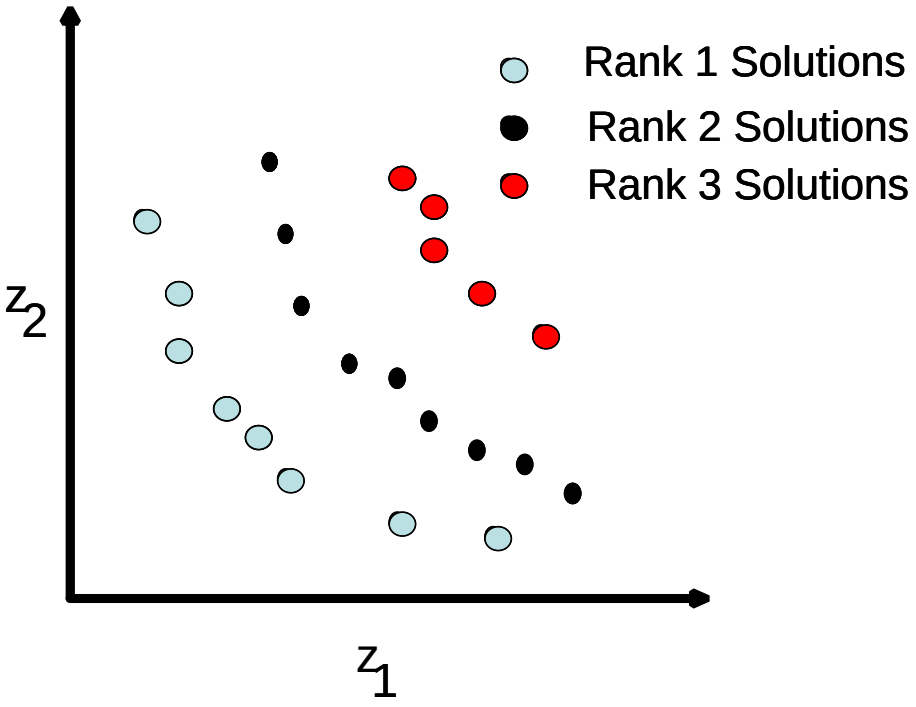
- Decision variables: scheduled start times to be assigned to *n* patients each day

- Goal: Generate the set of non-dominated schedules to understand tradeoffs between waiting and length of day

- Methods:
  - Schedules generated using a genetic algorithm (GA)
  - Non-dominated sorting used to identify the Pareto set and feedback into GA

Pareto Set

- Non-dominated sorting genetic algorithm of Deb *et al.*(2000) used to rank schedules



Selection Procedure

- Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) used to determine if solution *i* “dominates” *j*

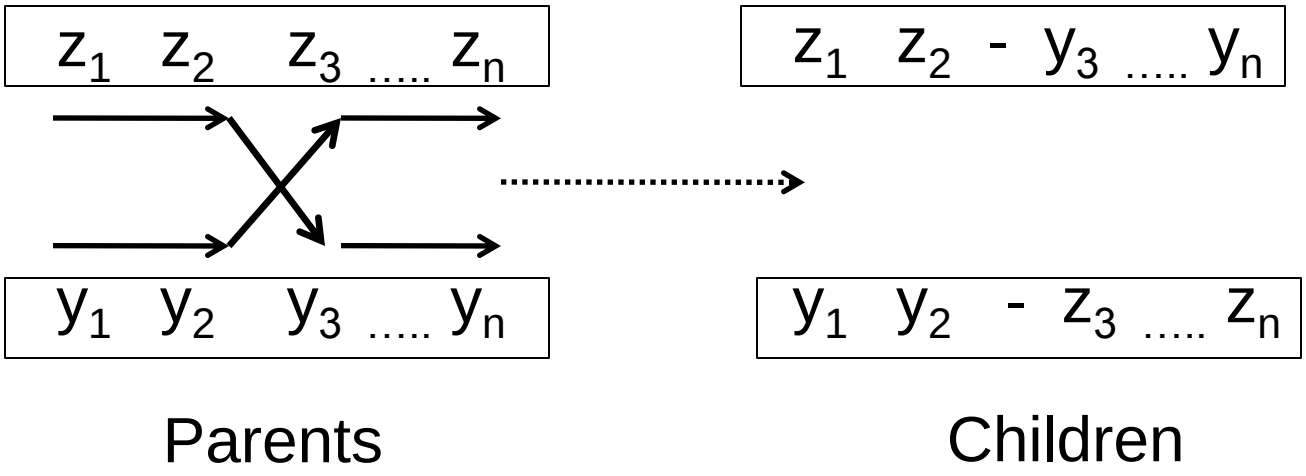
- Solution *i* “dominates” *j* if:

and

$$E[W_i] < E[W_j] \qquad\qquad E[L_i] < E[L_j]$$

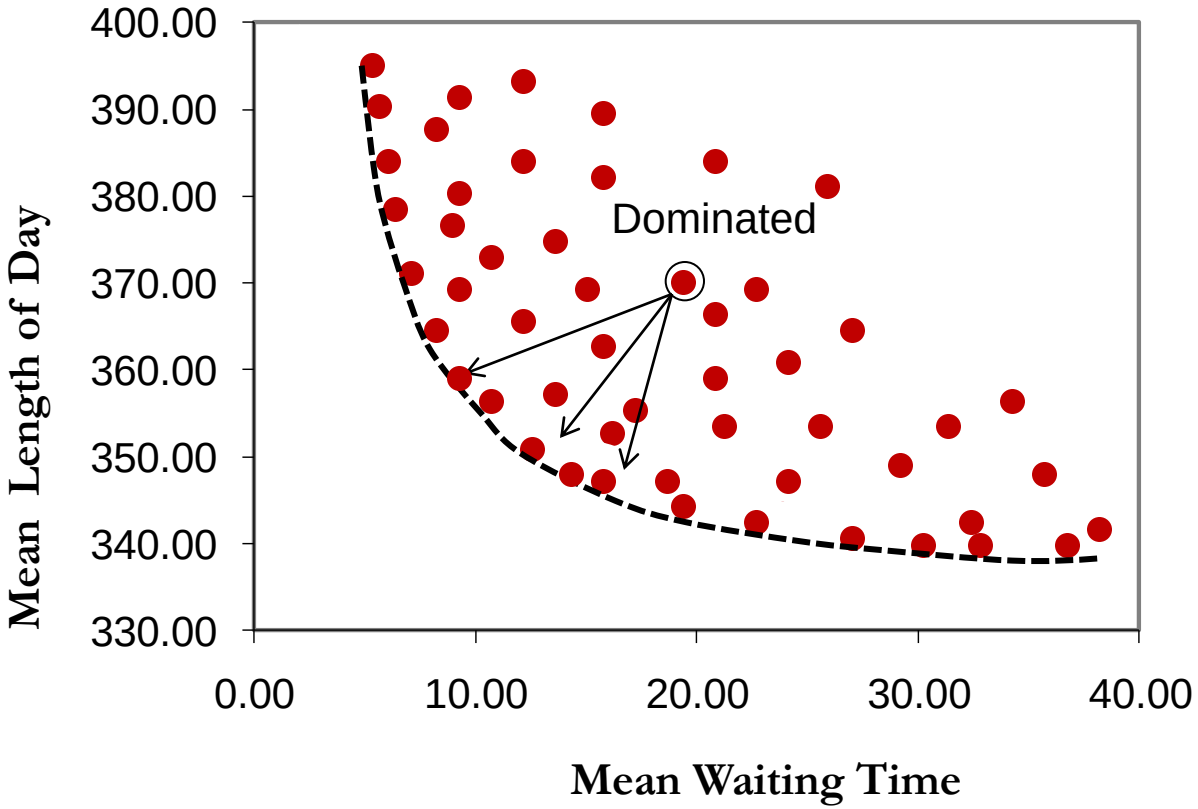
Genetic Algorithm

- Main features of the GA:
  - Randomly generated initial population of schedules
  - Selection based on 1) ranks and 2) crowding distance
  - Single point crossover:



  - Mutation

Appointment Schedules



Conclusions

- Simple “hedging” heuristic is nearly as good as more complicated genetic algorithm:
  - Set initial appointment time to start of day:

$$a_i = 0$$

  - Set remaining appointment times incrementally

$$a_i = a_{i-1} + \mu + \Delta$$

Current and Future Research

- Dynamic (online) scheduling problems
- Study of no-shows and overbooking
- Accounting for rescheduling during the day
- Planning and scheduling of chemotherapy treatment centers

NC STATE UNIVERSITY

Questions?

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