Optimization of On-line Appointment Scheduling

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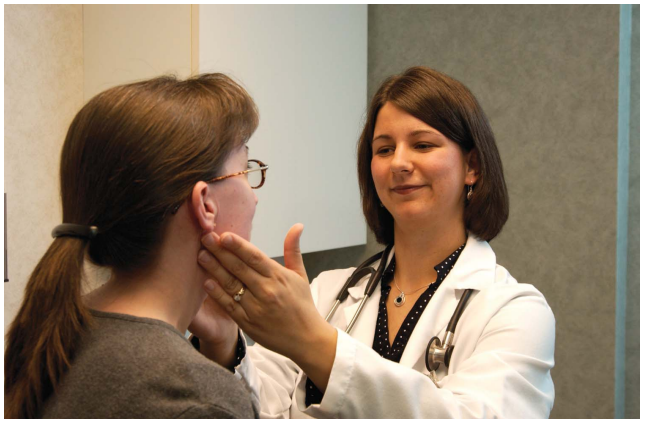
Appointment Scheduling Systems

- Interface between healthcare providers and patients
- Arises in many healthcare contexts
  - Primary care
  - Radiation Oncology
  - Surgery
  - Outpatient Procedures
  - Chemotherapy

Scheduling Challenges

- Competing criteria
  - Patient waiting time
  - Provider idle time and overtime
- Complicating Factors
  - Uncertain service durations
  - Uncertain patient demand
    - No-shows
    - Urgent Add-ons



Research Questions

Given a probabilistic arrival process for customer appointment requests to a single server, in which appointments must be quoted on-line:

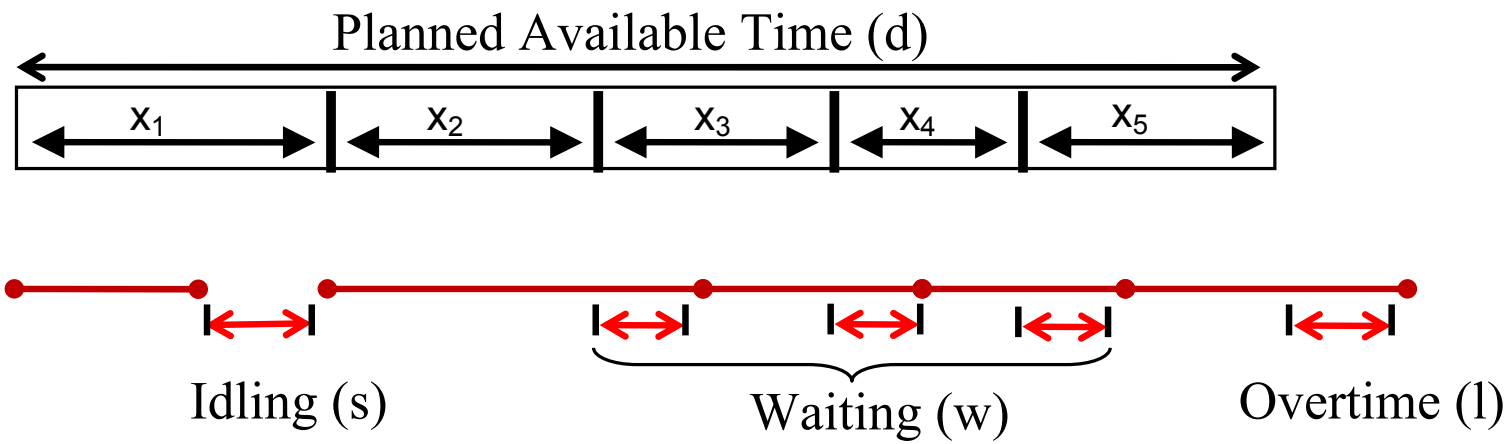
- What is the structure of the optimal appointment schedule?
- How can problems be classified into easy and hard?
- How important is it to find optimal schedules?

Presentation Outline

- Introduction
- Problems
  - Static Appointment Scheduling
  - Dynamic Appointment Scheduling
  - Dynamic Appointment Sequencing and Scheduling
- Conclusions
- Other Research

Static Appointment Scheduling Problem

Problem: Schedule n customers with uncertain service times during a fixed length of day, d



Common Heuristics

Mean Service Times:

$$\begin{aligned} a_1 &= 0 \\ a_i &= a_{i-1} + \mu_{i-1}, \quad \forall i \end{aligned}$$

Hedging:

$$\begin{aligned} a_1 &= 0 \\ a_i &= a_{i-1} + \mu_{i-1} + \kappa\sigma_{i-1}, \quad \forall i \end{aligned}$$

Ho, C., H. Lau. 1992. Minimizing Total Cost in Scheduling Outpatient Appointments, *Management Science* 38(12).

Cayirli, T., E. Veral. 2003. Outpatient Scheduling in Health Care: A Review of Literature, *Production and Operations Management* 12.

Literature Review

Queuing Analysis

- Bailey and Welch (1952)
- Jansson (1966)
- Sabria and Daganzo (1989)

Heuristics

- White and Pike (1964)
- Soriano (1966)
- Ho and Lau (1992)

Optimization

- Weiss (1990)
- Wang (1993)
- Denton and Gupta (2003)

Two-Stage Stochastic Linear Program

- First stage decisions

  x_i: Time allowance for customer i

- Second stage decisions

  $w_i(\omega)$: Customer i waiting time
  $\ell(\omega)$: Server overtime w.r.t. length of session d

- Random service durations:

  $Z_i(\omega)$: Random service time for customer i

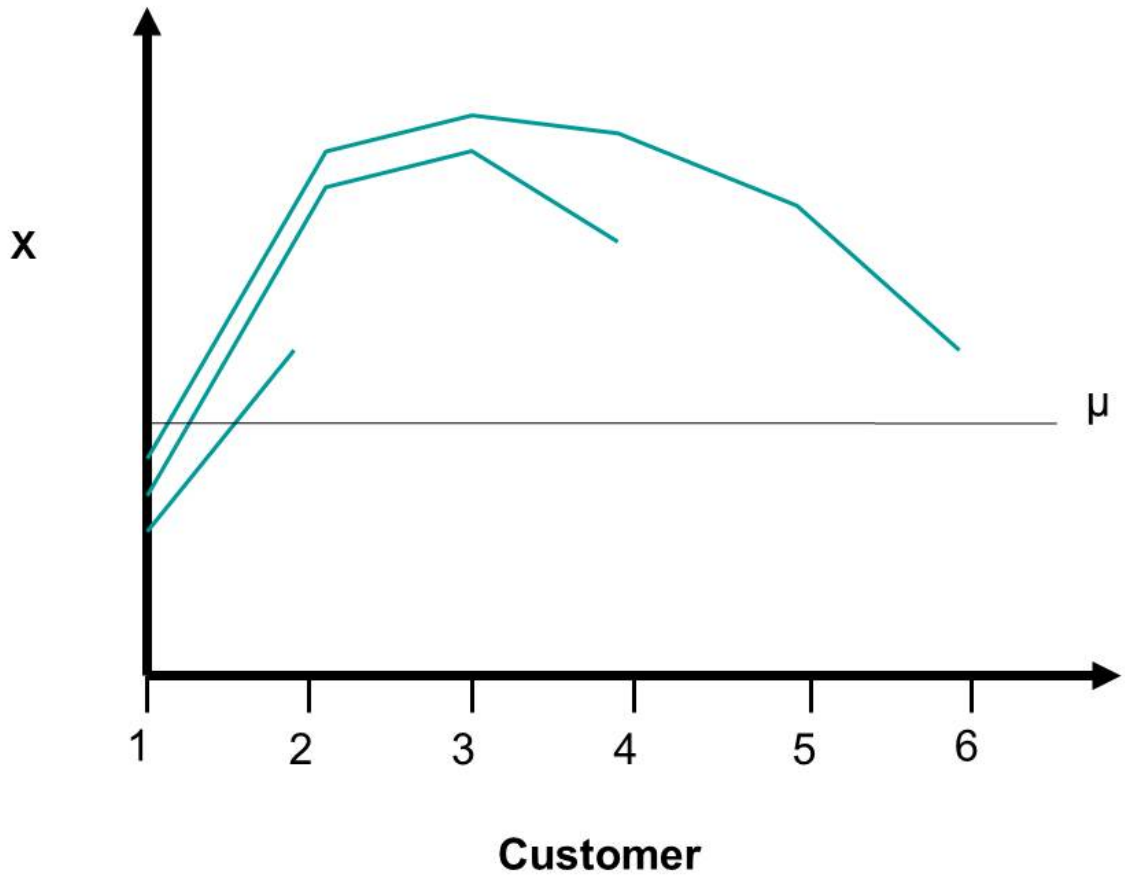
Model Formulation

$$\min E_\omega[\sum_{i=2}^{n} c_i^w w_i(\omega) + c^\ell \ell(\omega)]$$

$$\begin{aligned}
s.t. \quad & w_2(\omega) && \geq Z_1(\omega) - x_1, \forall \omega \\
& - w_2(\omega) + w_3(\omega) && \geq Z_2(\omega) - x_2, \forall \omega \\
& \ddots \quad \ddots && \vdots \\
& - w_{n-1}(\omega) + w_n(\omega) && \geq Z_{n-1}(\omega) - x_{n-1}, \forall \omega \\
& - w_n(\omega) + \ell(\omega) && \geq Z_n(\omega) + \sum_{i=1}^{n-1} x_i - d, \forall \omega
\end{aligned}$$

$$\mathbf{x} \geq \mathbf{0}, \ \mathbf{w}(\omega), \ell(\omega) \geq \mathbf{0}, \forall \omega$$

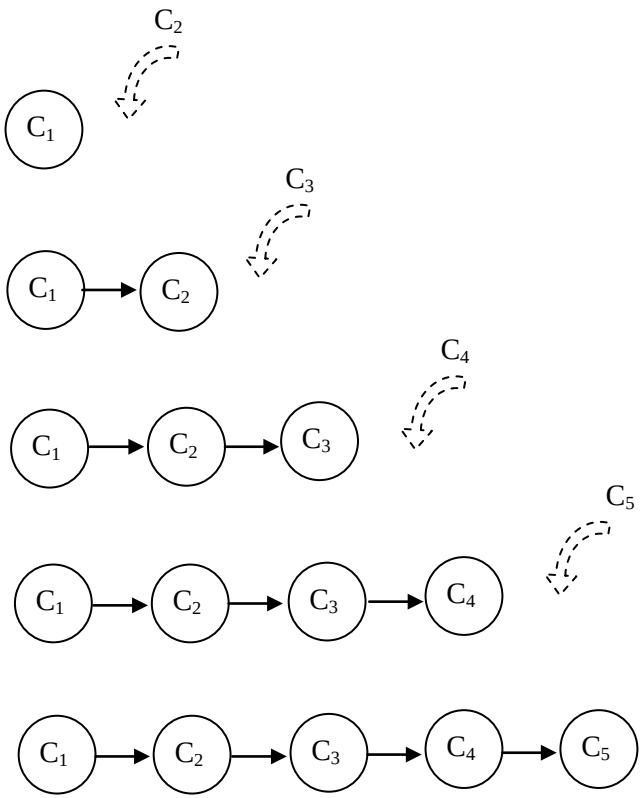
Example: 6 Customers



Denton, B.T. and Gupta D., 2003, "A Sequential Bounding Approach for Optimal Appointment Scheduling," *IIE Transactions*, 35, 1003-1016

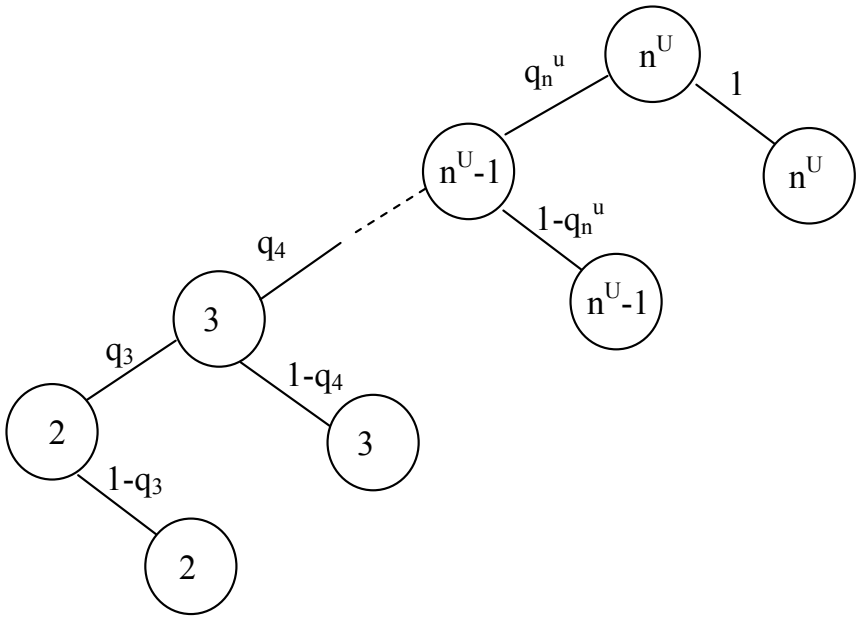
Dynamic Appointment Scheduling

Problem: Up to n^U customers are scheduled dynamically as they request appointments. Appointment requests are probabilistic.



Multi-stage Stochastic Program

Appointment requests are defined by a multi-stage scenario tree:



$$\min_{x_1}\{(1-q_3)Q_2(x_1)+\min_{x_2}\{q_3(1-q_4)Q_3(x_2)+\cdots+\min_{x_{n^U-1}}\{(\prod_{i=3}^{n^U})(q_i)Q_{n^U}(x_{n^U-1})\}\cdots\}\}$$

Model Formulation: Stage j

$$
\begin{aligned}
Q_j(x_j, \omega_j) \quad = \quad & \min_{\mathbf{w},\ell} \{ \sum_{i=2}^{j+1} c_i^w w_{j,i}(\omega_j) + c^\ell \ell_{j+1}(\omega_j) \} \\
s.t \quad & w_{j,2}(\omega_j) && \geq Z_1(\omega_j) - x_1 \\
& -w_{j,2}(\omega_j) + w_{j,3}(\omega_j) && \geq Z_2(\omega_j) - x_2 \\
& \quad \ddots \quad \ddots && \quad \vdots \\
& -w_{j,j}(\omega_j) + w_{j,j+1}(\omega_j) && \geq Z_j(\omega_j) - x_j \\
& -w_{j,j+1}(\omega_j) + \ell_{j+1}(\omega_j) && \geq Z_{j+1}(\omega_j) + \sum_{i=1}^{j} x_i - d \\
& w_{j,i}(\omega_j) \geq 0 \ \forall i, \ell_j(\omega_j) \geq 0.
\end{aligned}
$$

Model Properties

Motivation for first come first serve (FCFS) appointment sequence:

Proposition

For $n^U = 2$ with i.i.d. service durations, and identical waiting costs, the optimal sequence is FCFS.

Counter-examples exist for non i.i.d. and nonidentical waiting costs.

Solution Methods

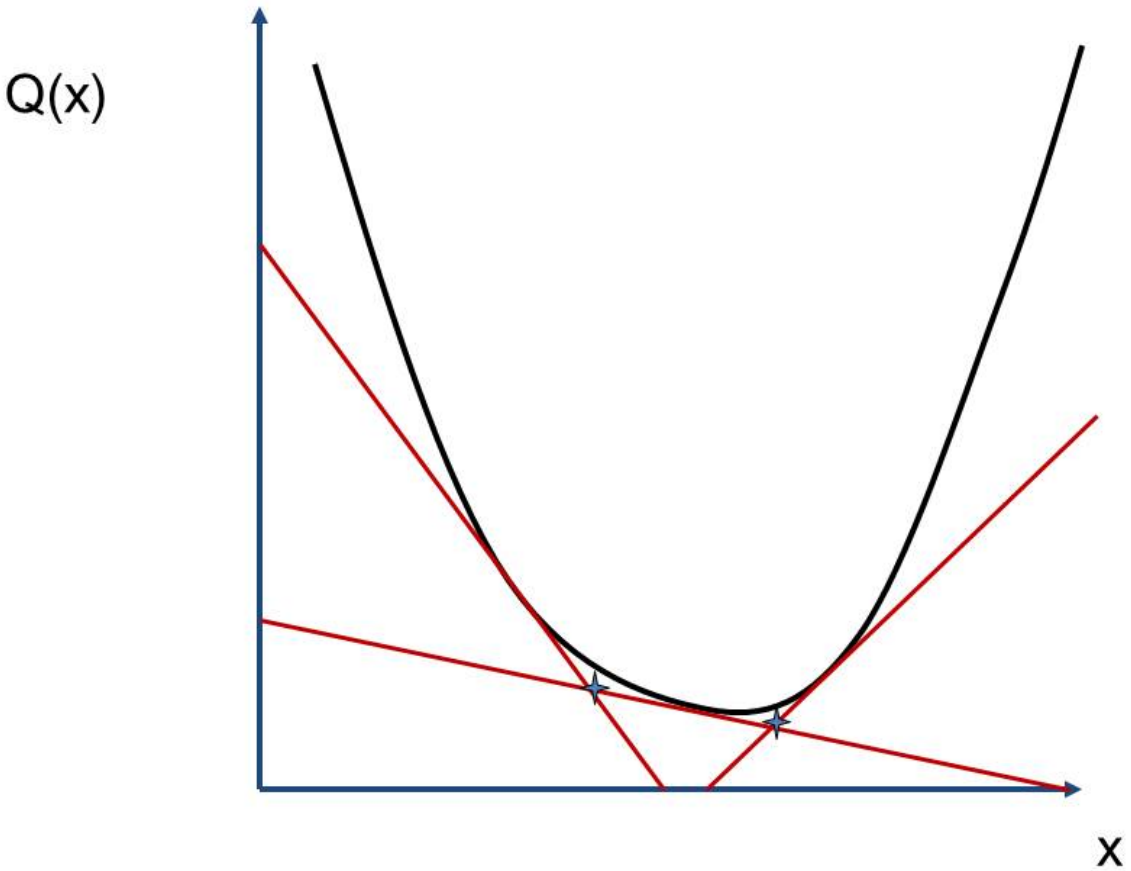
Variants of *nested decomposition*:

- Fast-forward-fast-back implementation
- Multi-cut method
- 2 variable method for master problems
- Valid inequalities based on relaxations of the mean value problem

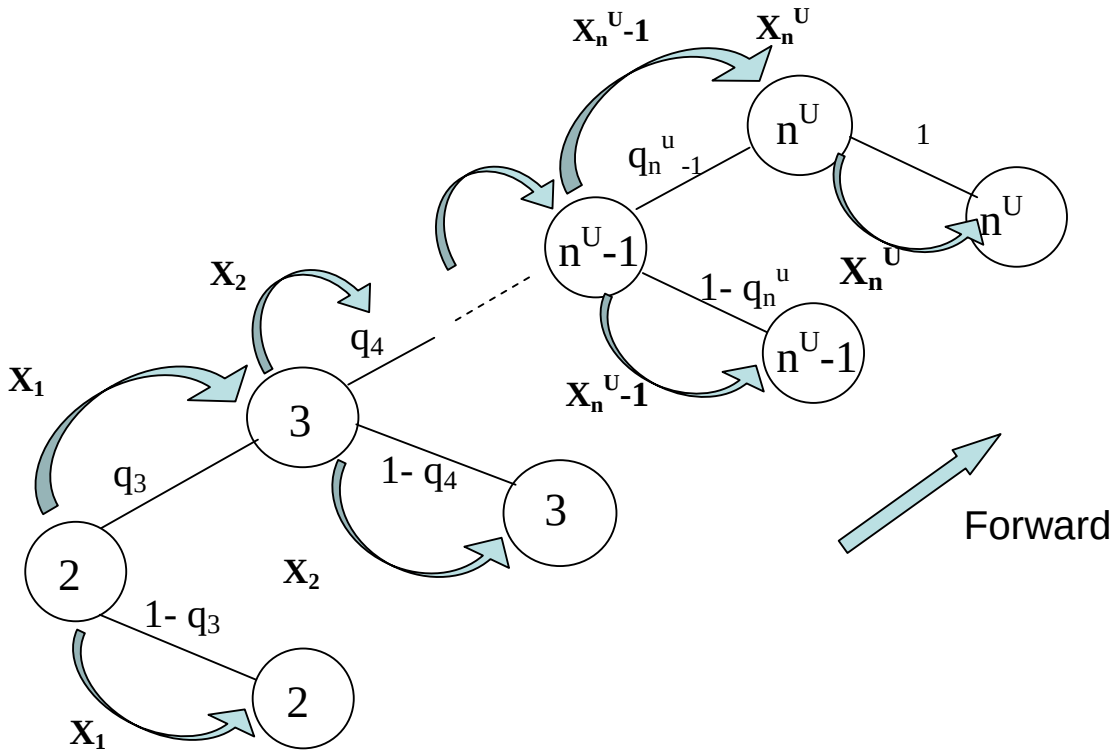
Outer Linearization

Outerlinearize the recourse function:

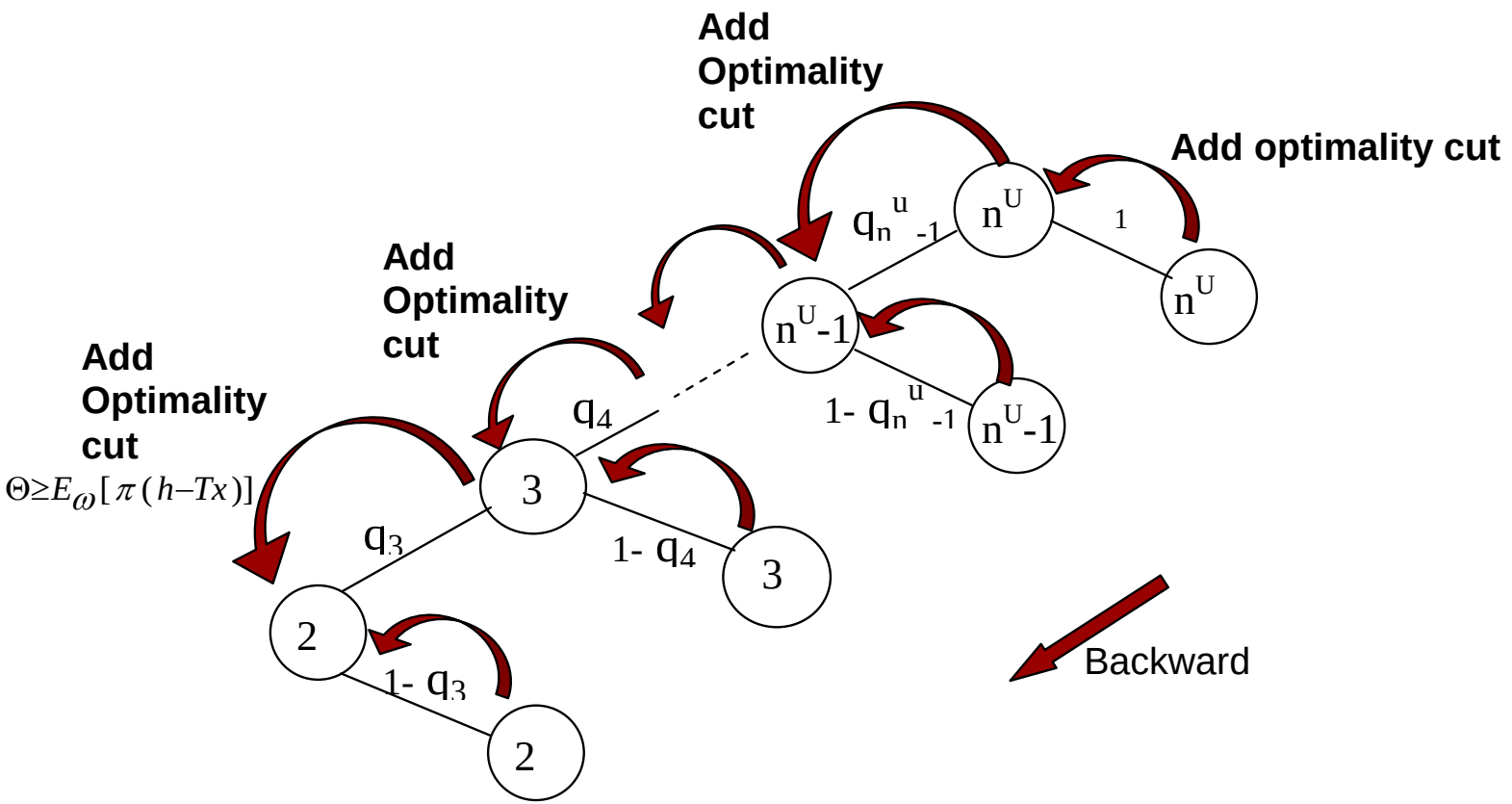
$$\min\{\theta \mid \theta \geq Q(x)\}$$



Methodology: Nested Decomposition Method

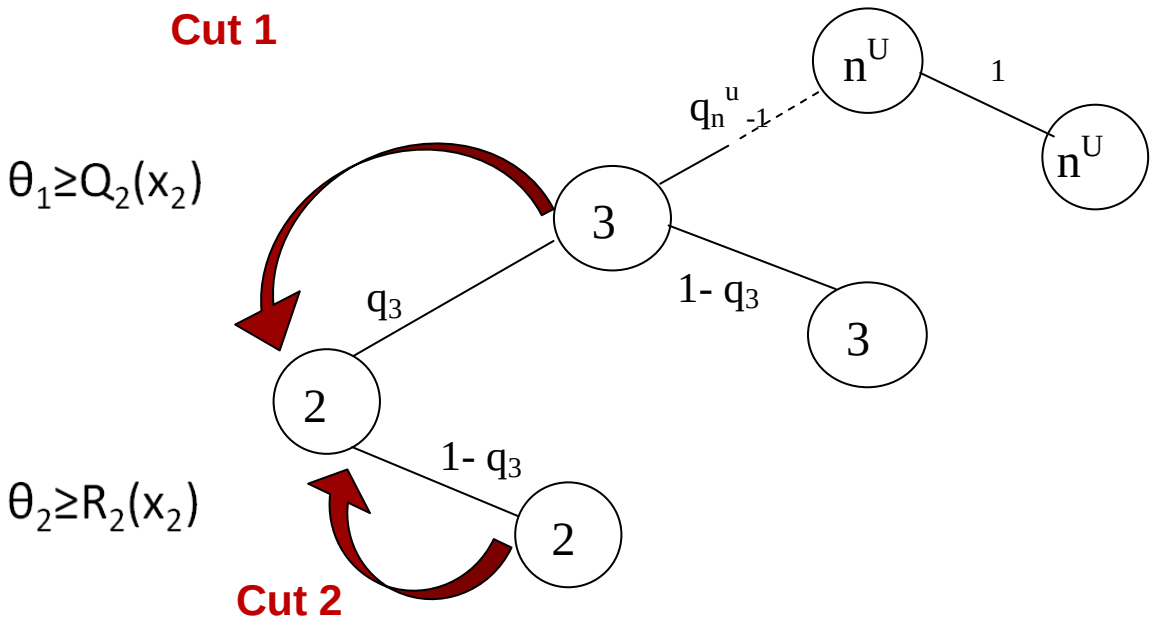


Methodology: Nested Decomposition Method



Multi-Cut Method

Separate cuts from master problems and subproblems (similar to multi-cut approach proposed by Birge and Louveaux (1985)

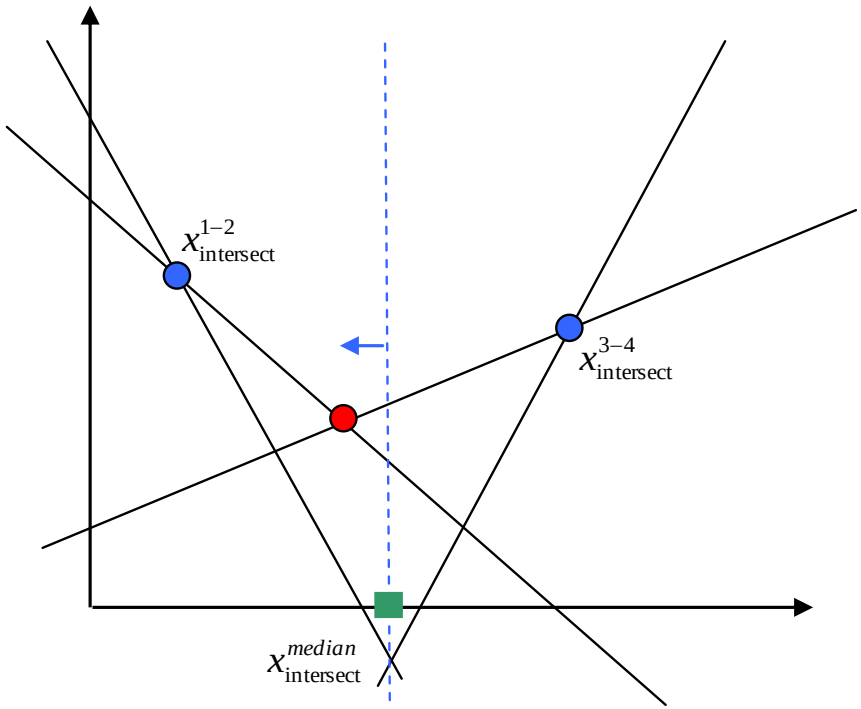


Two-variable LPs

- Master problems at each stage are two-variable LPs (x_j and θ_j)

$$\alpha_j x_j + \theta_j \geq \beta - (\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_{j-1} x_{j-1})$$

- Solve LPs with a modified version of the algorithm proposed by Dyer (1984)



Valid Inequalities

Proposition

The optimal solution to the mean value problem is $\bar{x}_i = \mu_i, \forall i$.

- Constraints based on mean value problem

$$\theta_j \geq Q_j(x, \bar{\xi})$$

- Similar to valid inequalities proposed by Batun et al. (2011)

Solution Methods

Several adaptations of *nested decomposition* were compared:

- Standard nested decomposition (ND)
- Multi-cut ND
- Two-variable ND
- ND with mean value valid inequalities (VI)

Comparisons of Methods

		Number of Iterations			CPU Time (seconds)		
		$n^U = 10$ (d=200)	$n^U = 20$ (d=400)	$n^U = 30$ (d=600)	$n^U = 10$ (d=200)	$n^U = 20$ (d=400)	$n^U = 30$ (d=600)
$\frac{c^\ell}{c^w} = 10$	ND	244	432	438	3.42	23.26	49.68
	Multi-cut ND	186	244	202	2.63	13.52	23.21
	Two-variable ND	254	406	362	3.56	24.06	43.59
	ND with VIs	232	370	442	3.65	20.83	51.79
$\frac{c^\ell}{c^w} = 1$	ND	192	330	392	2.75	16.77	42.46
	Multi-cut ND	106	184	174	1.55	9.81	19.85
	Two-variable ND	186	290	284	2.54	16.32	31.82
	ND with VIs	188	306	364	2.98	16.89	42.50
$\frac{c^\ell}{c^w} = 0.1$	ND	190	302	422	2.55	14.54	43.48
	Multi-cut ND	96	176	162	1.33	8.79	17.45
	Two-variable ND	186	290	384	2.37	15.49	42.95
	ND with VIs	174	284	412	2.62	14.86	45.70

2 QuadCore Intel® Xeon® Processor 2.50GHz CPU, 16GB Ram, CPLEX 11.0

Value of Stochastic Solution (VSS)

Table: VSS for test instances with $Z_i \sim U(20, 40)$ and $q_i = 0.5$ for add-on requests.

Number of Customers (Routine, Add-on)	VSS (%)		
	$d = 200$		
	$\frac{c^\ell}{c^w} = 10$	$\frac{c^\ell}{c^w} = 1$	$\frac{c^\ell}{c^w} = 0.1$
(0,30)	9.63	65.59	95.15
(10,30)	1.40	19.63	79.41
(20,30)	0.50	23.63	80.33

Example: Scheduling an Endoscopy Suite

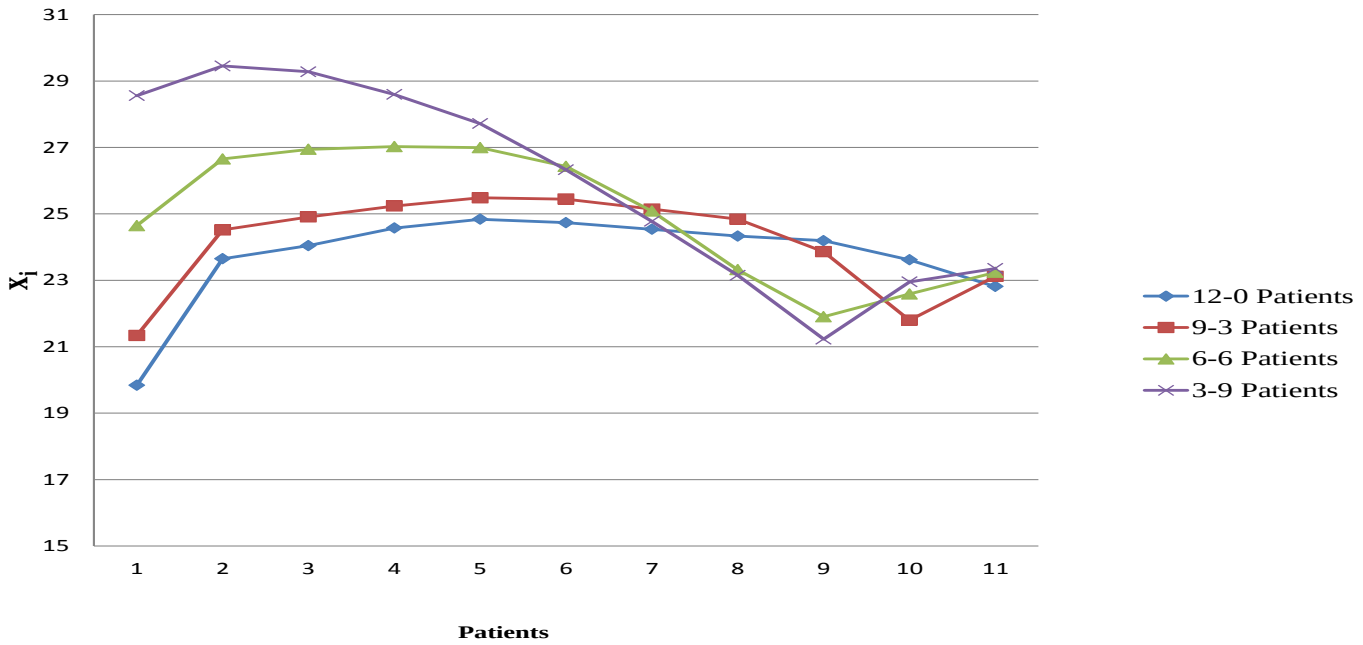


Figure: Service times based on colonoscopy times for an outpatient endoscopy practice: $Z_i \sim \text{Lognormal}(23.55, 11.89)$, $\forall i$.

Example: Multi-Procedure Room Endoscopy Practice

Endoscopy Practice:

- 2 intake rooms
- 2 procedure rooms
- 4 recovery rooms
- Service timed based on empirical data

Table: Expected waiting time and overtime according to different schedules

	Heuristic			Stochastic Program Based Schedule		
	$\frac{c^\ell}{c^w} = 10$	$\frac{c^\ell}{c^w} = 1$	$\frac{c^\ell}{c^w} = 0.1$	$\frac{c^\ell}{c^w} = 10$	$\frac{c^\ell}{c^w} = 1$	$\frac{c^\ell}{c^w} = 0.1$
Expected total cost	975.19	111.72	253.71	878.03	104.58	162.65
Expected waiting time	15.78			16.28	10.54	5.06
Expected overtime	95.94			86.17	94.05	111.97

Dynamic Appointment Sequencing and Scheduling

The **appointment request sequence** and the **appointment arrival sequence** are not necessarily the same.

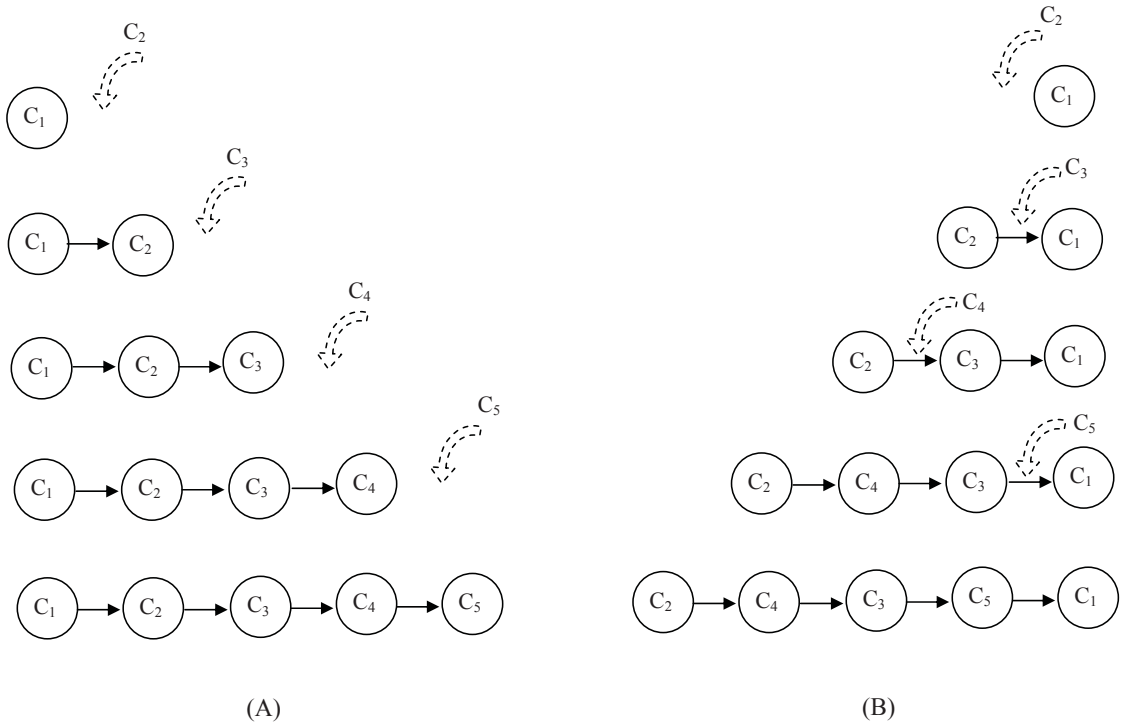


Figure: (A) FCFS; (B) Example of the general case.

Two Stage Stochastic Integer Program

Minimize {Cost of Indirect Waiting + E_ω[Direct Waiting + Overtime]}

First Stage Decisions:

- Customer sequencing (binary)
- Service time allowances (continuous, sequence dependent)
- Appointment times (continuous, sequence dependent)

Second Stage Decisions:

- Waiting time (continuous, sequence dependent)
- Overtime (continuous)

Two Stage Stochastic Integer Program

First Stage Decisions:

$o_{j,i,i'}$: binary sequencing variable where $o_{jii'} = 1$ if customer i immediately precedes i' at stage j, and $o_{ii'j} = 0$ otherwise

$x_{j,i,i'}$: time allowance for customer i given that i immediately precedes i' at stage j

$a_{j,i,i'}$: appointment time of customer i', given that i immediately precedes i' at stage j

Second Stage Decisions:

$w_{j,i,i'}(\omega)$: waiting time of customer i' given that customer i immediately precedes i' at stage j under duration scenario ω

$s_{j,i,i'}(\omega)$: server idle time between customer i and i', given that i immediately precedes i' at stage j

$\ell_j(\omega)$: overtime at stage j with respect to session length d

First Stage Problem

$$\min \sum_{j=1}^{n} p_j [\sum_{i=1}^{j} \sum_{i'=1}^{j} c^a_{i'} a_{j,i,i'}] + Q(\mathbf{o}, \mathbf{x})$$

s.t.

$$\sum_{i'=1}^{j+1} o_{j,i,i'} = 1, \quad \sum_{i'=0}^{j+1} o_{j,i',i} = 1 \qquad \forall j, i = 1, 2, \ldots, j$$

$$\sum_{i=0}^{j+1} \sum_{i=0}^{j+1} o_{j,i,i'} = j + 1 \qquad \forall j$$

$$o_{j,i,j} + o_{j,j,i'} - 2(o_{j-1,i,i'} - o_{j,i,i'}) \geq 0 \qquad \forall j, \ \forall i, i' < j$$

$$x_{j,i,i'} \leq M_1 o_{j,i,i'}, \quad a_{j,i,i'} \leq M_1 o_{j,i,i'} \qquad \forall j, i, i'$$

$$\sum_{i'=1}^{j+1} x_{j,i,i'} = \sum_{i'=1}^{j+1} a_{j,i,i'} - \sum_{i'=1}^{j+1} a_{j,i',i} \qquad \forall j, \ i$$

$$x_{j,i,i'}, \ a_{j,i,i'} \geq 0, \ o_{j,i,i'} \in \{0, 1\} \qquad \forall j, i, i', \forall j$$

Second Stage Subproblem

$$Q(\mathbf{o}, \mathbf{x}, \omega) = \min \quad E_\omega[\sum_{i=1}^{j}\sum_{i'=1}^{j}(c_{i'}^{w} w_{j,i,i'}(\omega) + c^{\ell}\ell_j(\omega)]$$

$s.t.$

$$w_{j,i,i'}(\omega) \le M_2(\omega) o_{j,i,i'} \qquad \forall i, i', j, \omega$$

$$s_{j,i,i'}(\omega) \le M_3(\omega) o_{j,i,i'} \qquad \forall i, i', j, \omega$$

$$-\sum_{i'=1}^{j} w_{j,i',i}(\omega) + \sum_{i'=1}^{j} w_{j,i,i'}(\omega) - \sum_{i'=1}^{j} s_{j,i,i'}(\omega) = Z_i(\omega) - \sum_{i'=1}^{j} x_{j,i,i'} \qquad \forall i, j, \omega$$

$$\ell_j(\omega) \ge \sum_{i=1}^{j}\sum_{i'=1}^{j} s_{j,i,i'}(\omega) + \sum_{i=1}^{j} Z_i(\omega) + \sum_{i'=1}^{j} x_{j,0,i'} - d \qquad \forall j, \omega$$

$$w_{j,i,i'}(\omega), s_{j,i,i'}(\omega), \ell_j(\omega) \ge 0, \qquad \forall j, i, i', \omega.$$

Model Properties

The addition of indirect waiting costs results in conditions under which FCFS is not optimal:

Proposition

For $n^U = 2$ with i.i.d. service durations if

$$c_2^a \geq c_1^w$$

then the optimal sequence is LCFS.

Methodology

Compared *L-shaped method* and *Integer L-shaped method*

- Fast solution to second stage subproblems
- Presolve
- Warm start
- Branch-and-cut vs. dynamic search
- MIP cuts (MIR, implied bound cuts, etc.)
- Mean value problem based valid inequalities

Computational Performance

No. of Customers	Class	Type of Customers	L-Shaped Method			
			CPU Time		# of Iterations	
			Average	Max	Average	Max
5	2.1	5 Add on Customers	449	484	192.9	202
	2.2	3 Routine + 2 Add on	2247.71	2546	608.7	660
7	2.3	7 Add on Customers	15000*	15000*	283	290
	2.4	4 Routine 3 Add on	15000*	15000*	241	247
10	2.5	10 Add on Customers	15000*	15000*	92	97
	2.6	7 Routine 3 Add on	15000*	15000*	93	102

Computational Performance

Table: Gap at the time of termination for the instances that are not solved to optimality

Problem Size	Instance No	Patient Type	Best Gap	
			L-Shaped Method	L-Shaped Method (mean value based cuts)
7 (uniform)	2.3	7 Add on Patients	107.12%	optimal
	2.4	4 Routine 3 Add on	174.62%	1.95%
10 (uniform)	2.5	10Add on Patients	240.11%	7.26%
	2.6	7 Routine 3 Add on	375.32%	1.99%
7 (lognormal)	3.3	7 Add on Patients	223.32%	21.99%
	3.4	4 Routine 3 Add on	335.02%	15.71%
10 (lognormal)	3.5	10Add on Patients	338.37%	31.53%
	3.6	7 Routine 3 Add on	517.307%	13.07%

Example 1: Structure of the Optimal Solution

Table: Examples with varying direct/indirect cost for instance 3.6 (7 routine, 3 add on, lognormal service times) parameters

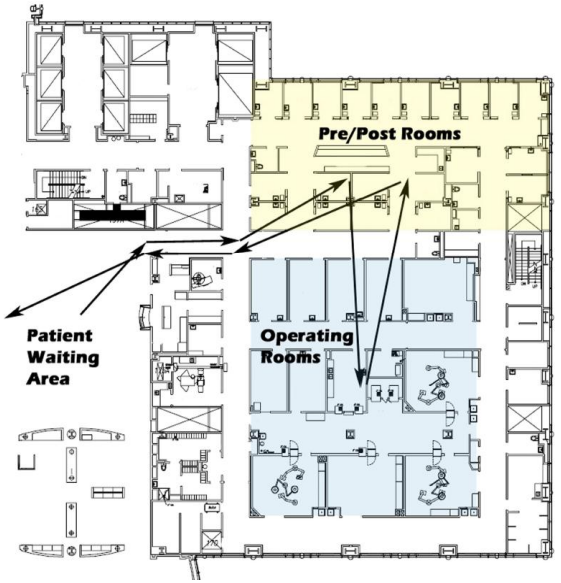
Instance No	c^a Routine	c^w Routine	c^a Add-on	c^w Add-on	c^L	Optimal Sequence	CPU Time		# of Iterations	
							ave	max	ave	max
1	0	1	0.1	0.1	10	R-R-R-R-R-R-R-A-A-A	12295.5	14980	55.2	598
2	0	1	10	10	10	A-A-A-R-R-R-R-R-R-R	1174.8	1852	163.5	209
3	0	1	50	50	10	A-A-A-R-R-R-R-R-R-R	418.2	613	94.9	122
4	0	1	100	100	10	A-A-A-R-R-R-R-R-R-R	257.6	522	67.4	112
5	0	1	250	250	10	A-A-A-R-R-R-R-R-R-R	117.2	290	36	73
6	0	1	500	500	10	A-A-A-R-R-R-R-R-R-R	52.5	112	18.1	36
7	0	1	750	750	10	A-A-A-R-R-R-R-R-R-R	28.1	48	10.3	17
8	0	1	1000	1000	10	A-A-A-R-R-R-R-R-R-R	19.4	30	7.1	10

Conclusions

- VSS can be as high as 95% and as low as 0.5%
- Large instances of dynamic scheduling problem can be solved efficiently but sequencing and scheduling is much harder
- FCFS generally optimal when probabilities of add-on customers are low and/or indirect cost of waiting is low
- Placement of add on customers is frequently “all or nothing”

Other Research

- Complex service systems with multiple servers and stages of service
- Uncertain service time, demand, and patient/provider behavior



Applications:

- Hospital surgery practices
- Outpatient procedure and treatment centers

Questions?

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