Stochastic Optimization for Scheduling in Healthcare Delivery Systems

Optimization Days
Canadian Healthcare Optimization Workshop
Montreal
May 10, 2017

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Summary

Optimization in Healthcare

Surgery Scheduling Examples:

- Example 1: Single OR scheduling
- Example 2: Multi-OR scheduling
- Example 3: Bi-criteria scheduling of multi-stage surgery suite

Wrap-up

Optimization in Healthcare

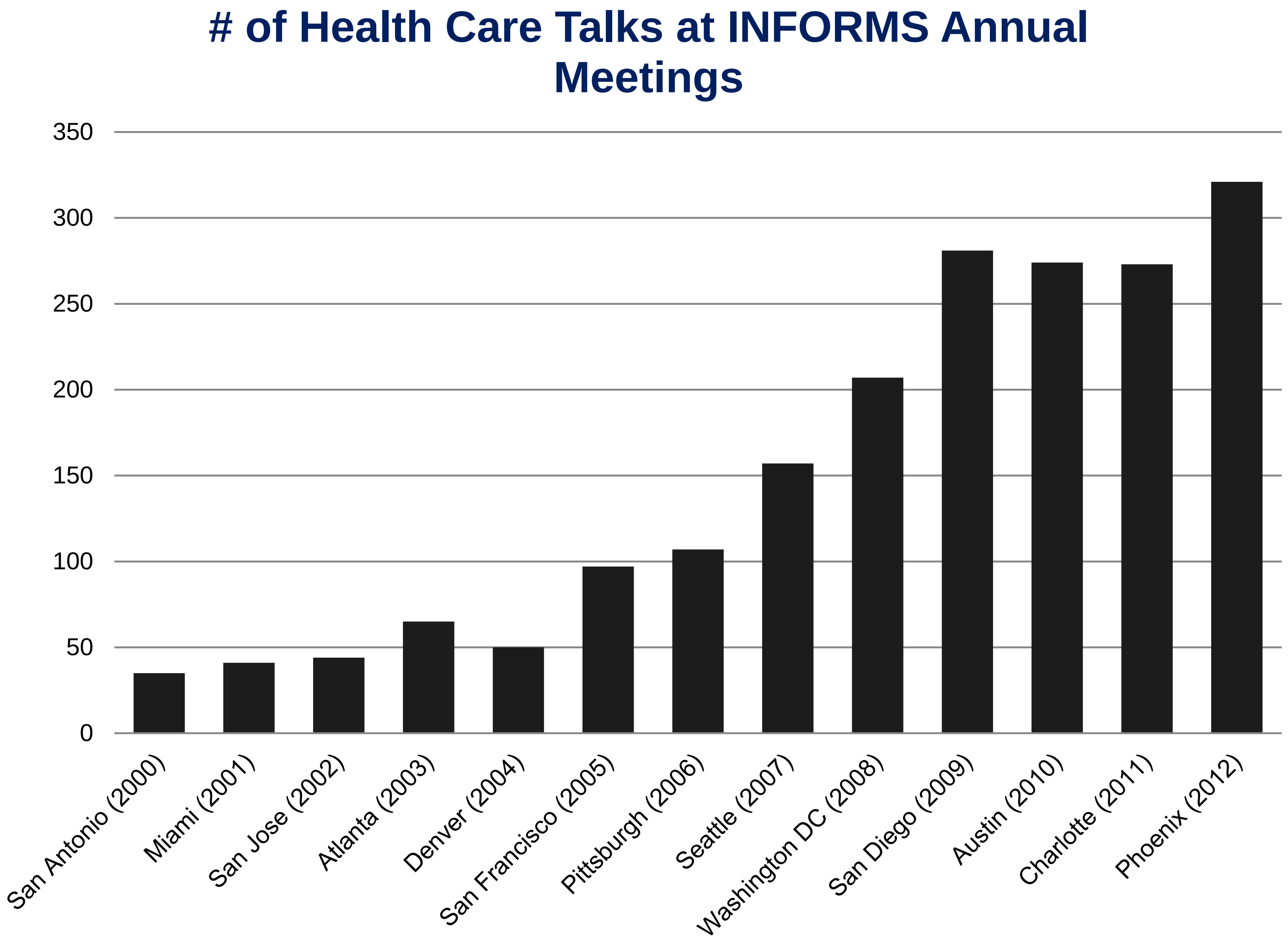
Nurse Scheduling

Ambulance Dispatching

Primary Care Panels

Inventory Management

Healthcare in Optimization



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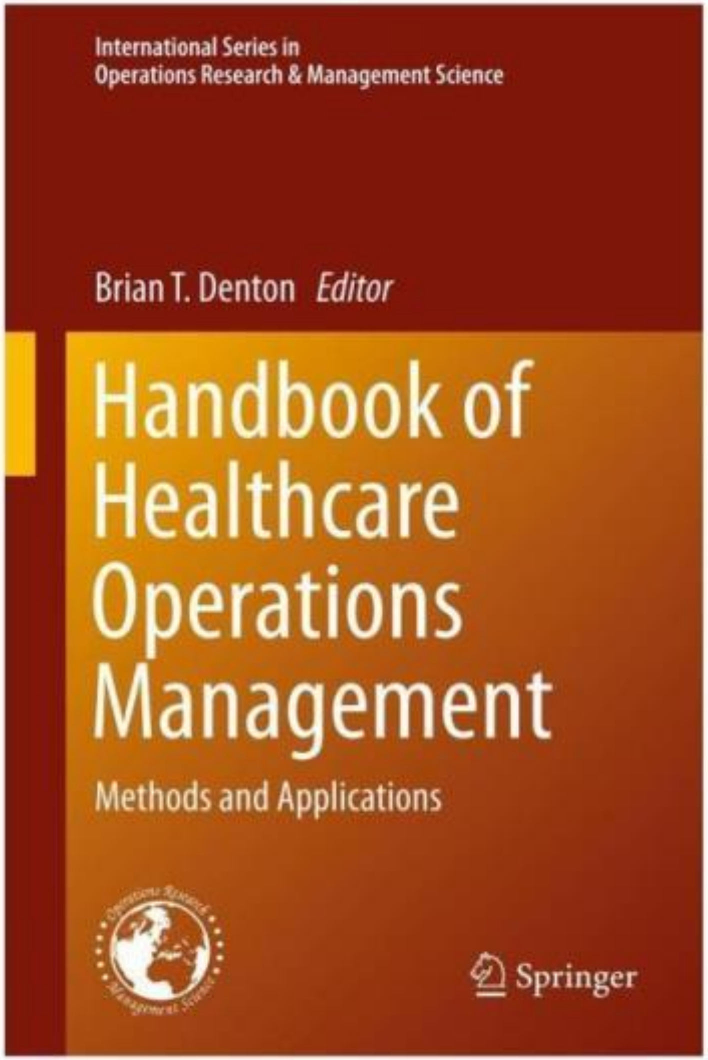

International Series in Operations Research & Management Science

Brian T. Denton Editor

Handbook of Healthcare Operations Management

Methods and Applications

Springer

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Surgical Care Delivery

- Efficient access to surgery is important for patient health and safety

- Surgery accounts for the largest proportion a hospital’s expenses and revenues

Surgery in the U.S.

- Hospitals
    - Open 24 hours a day
    - Patients recover in the hospital
    - Handle complex surgeries

- Ambulatory Surgery Centers
    - Normally open 7am to 5pm
    - Patients admitted and discharged same day
    - Lower cost and lower infection rate than hospitals

Surgery Process


Intake

Surgery

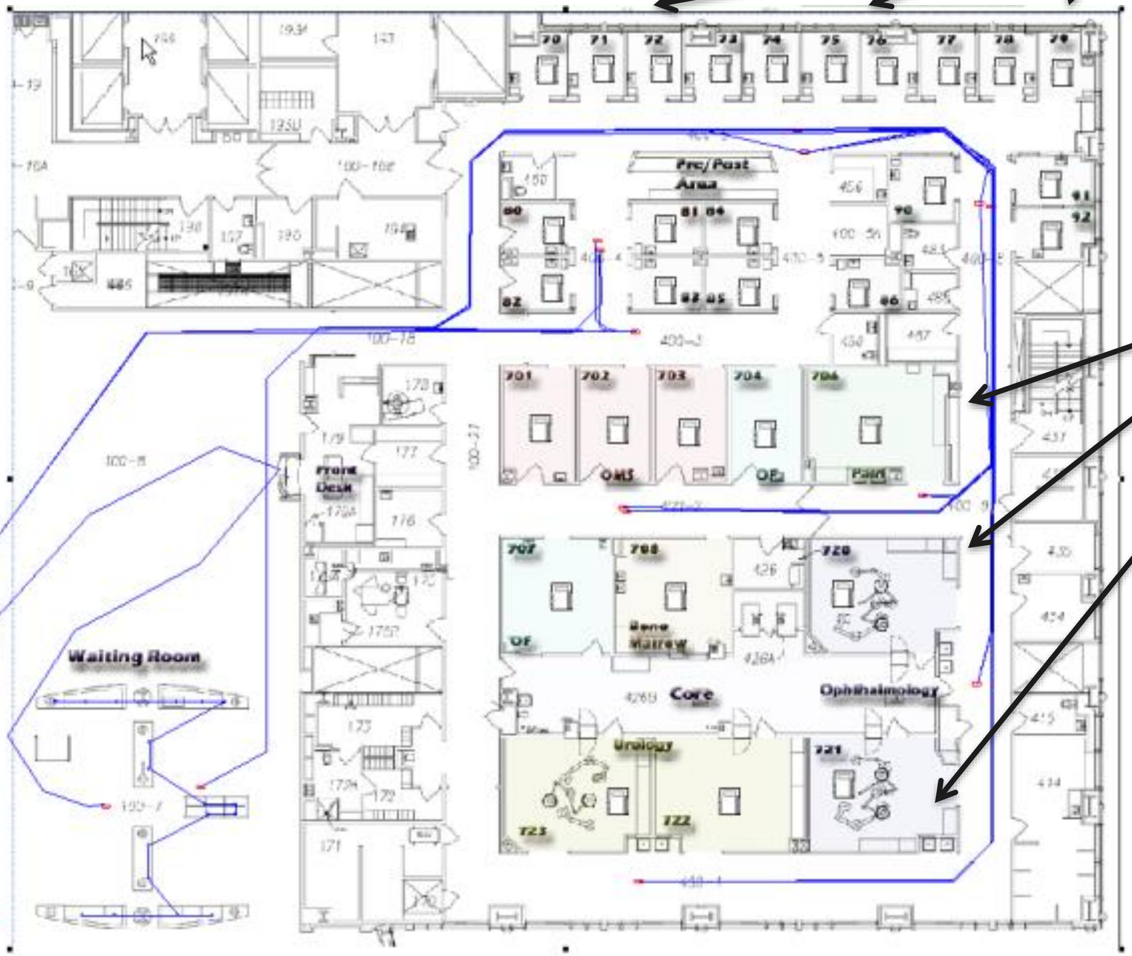
Recovery

Patient Intake: administrative activities, pre-surgery exam, gowning, site prep, anesthetic

Surgery: incision, one or multiple procedures, pathology, closing

Recovery: post anesthesia care unit (PACU)

Ambulatory Surgery Center Blueprint



Pre/post rooms

Operating rooms

Patient waiting area

Blue lines represent patient flow

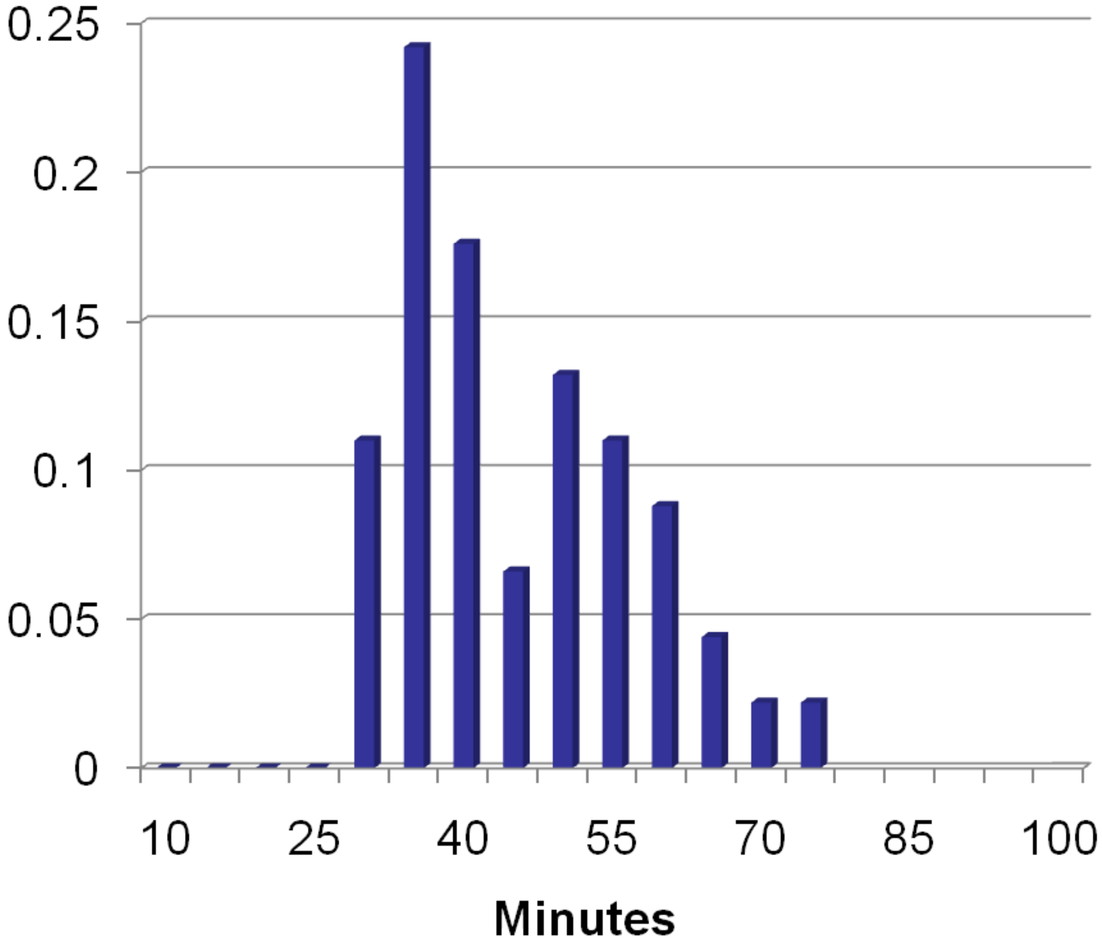
Management decisions that can be supported with optimization models

- Surgery start time scheduling
- Number of ORs and staff to activate each day
- Surgery-to-OR assignment decisions
- Scheduling of staff in intake, surgery, and recovery

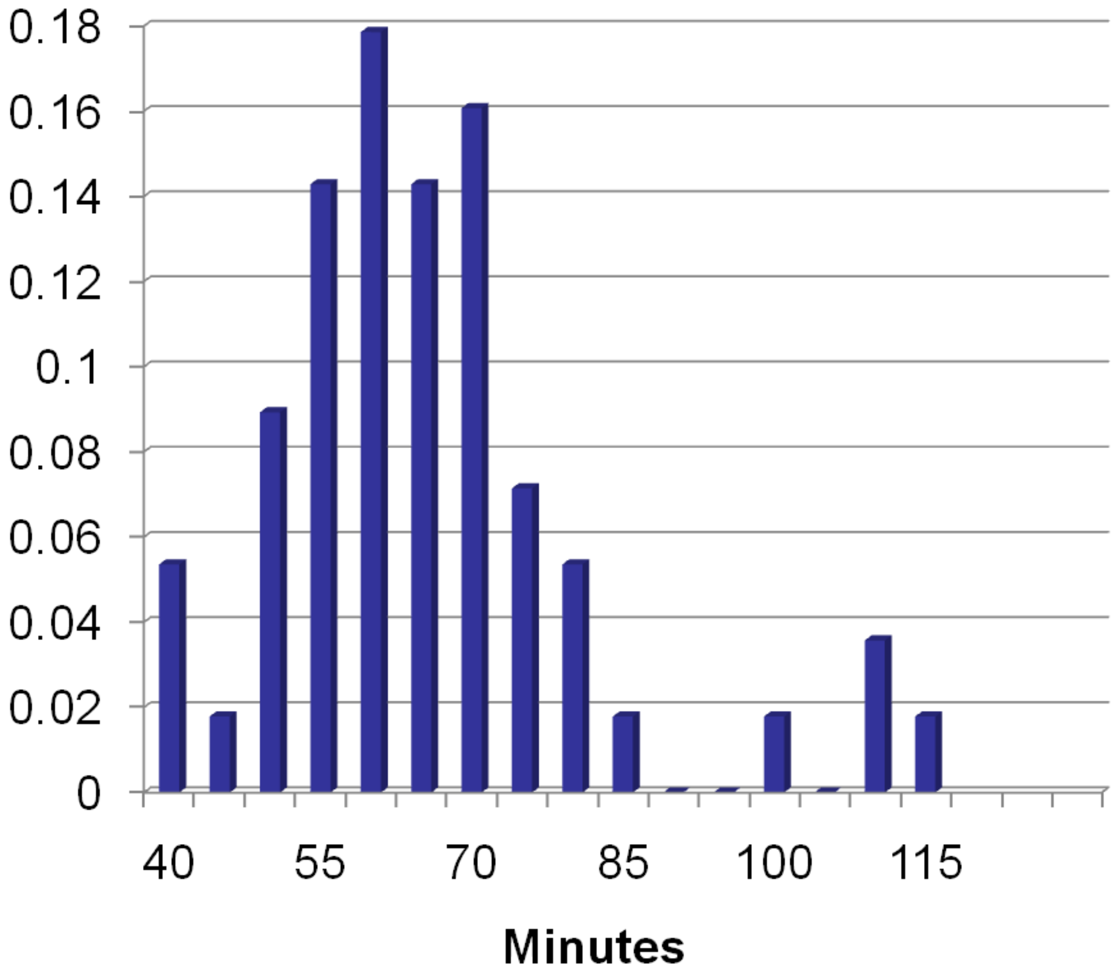
Complicating Factors

- High cost of resources and fixed time to complete activities
- Large number of activities to be coordinated in a highly constrained environment
- Uncertainty in duration of activities
- Multiple competing criteria

Empirical distribution for tonsilectomy



Empirical distribution for hernia repair



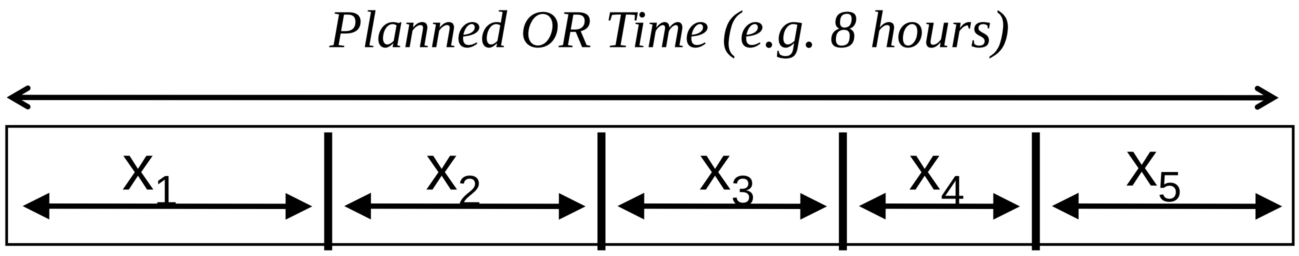
Example 1
Single Operating Room (OR) Scheduling

Single OR Scheduling Problem

For a single OR find the optimal time to allocate for each surgery to minimize the cost of:

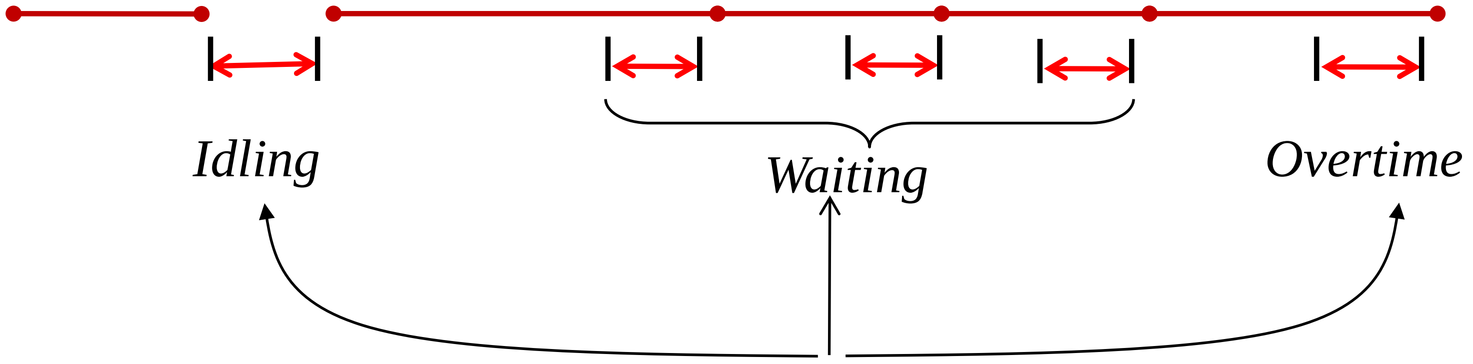
- Patient and surgery team waiting
- Unutilized (idle) time of the operating room
- Overtime

Single OR Scheduling



Planned OR Time (e.g. 8 hours)

Example Scenario:



Idling

Waiting

Overtime

Goal: Min{ Idling + Waiting + Overtime}

Stochastic Optimization Model

$$\min_{\boldsymbol{x}}\{\overbrace{\sum_{i=1}^{n} C_i^{w} E_{\mathbf{Z}}[W_i]}^{\text{Cost of Waiting}} + \overbrace{\sum_{i=1}^{n} C^{s} E_{\mathbf{Z}}[S_i]}^{\text{Cost of Idling}} + \overbrace{C^{L} E_{\mathbf{Z}}[L]}^{\text{Cost of Overtime}}\}$$

Random surgery time

Planned time for surgery

$$W_i = \max(W_{i-1} + Z_{i-1} - x_{i-1}, 0)$$

$$S_i = \max(-W_{i-1} - Z_{i-1} + x_{i-1}, 0)$$

$$L = \max(W_n + Z_n + \Sigma x_i - T, 0)$$

Literature Review – Single Server

Queuing Analysis:

- Mercer (1960, 1973)
- Jansson (1966)
- Brahimi and Worthington (1991)

Assumes steady state is reached, i.i.d. service times, fix time allotment

Heuristics:

- White and Pike (1964)
- Soriano (1966)
- Ho and Lau (1992)

No guarantee of optimal solution

Optimization:

- Weiss (1990) – 2 surgery news vendor model
- Wang (1993) – Exploited phase type distribution property
- Denton and Gupta (2003) – General stochastic programming formulation

Reformulation as a Stochastic Program

$$\min\{ E_Z[\sum_{i=2}^{n} c_i^w w_i + \sum_{i=2}^{n} c^s s_i + c^L l]\}$$

s.t.

$$\begin{array}{lllll}
w_2 & & - s_2 & & = Z_1 - x_1 \\
- w_2 + w_3 & & & - s_3 & = Z_2 - x_2 \\
& \ddots & & \ddots & \\
& - w_n & & - s_n + l - g & = Z_n - d + \sum_{j=1}^{n-1} x_i
\end{array}$$

$$x_i \geq 0, w_i \geq 0, s_i \geq 0, i = 1,\ldots, n, \quad l, g \geq 0$$

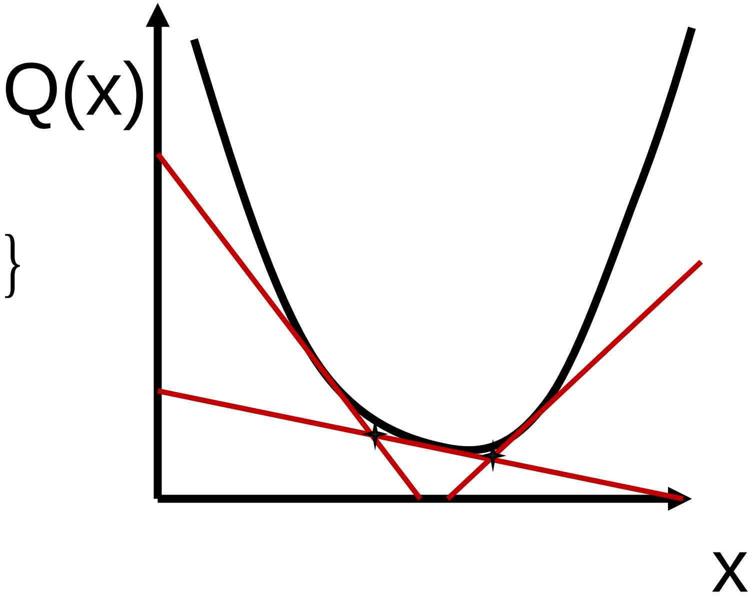
Two Stage Recourse Problem

Initial Decision (x) → Uncertainty Resolved → Recourse (y)

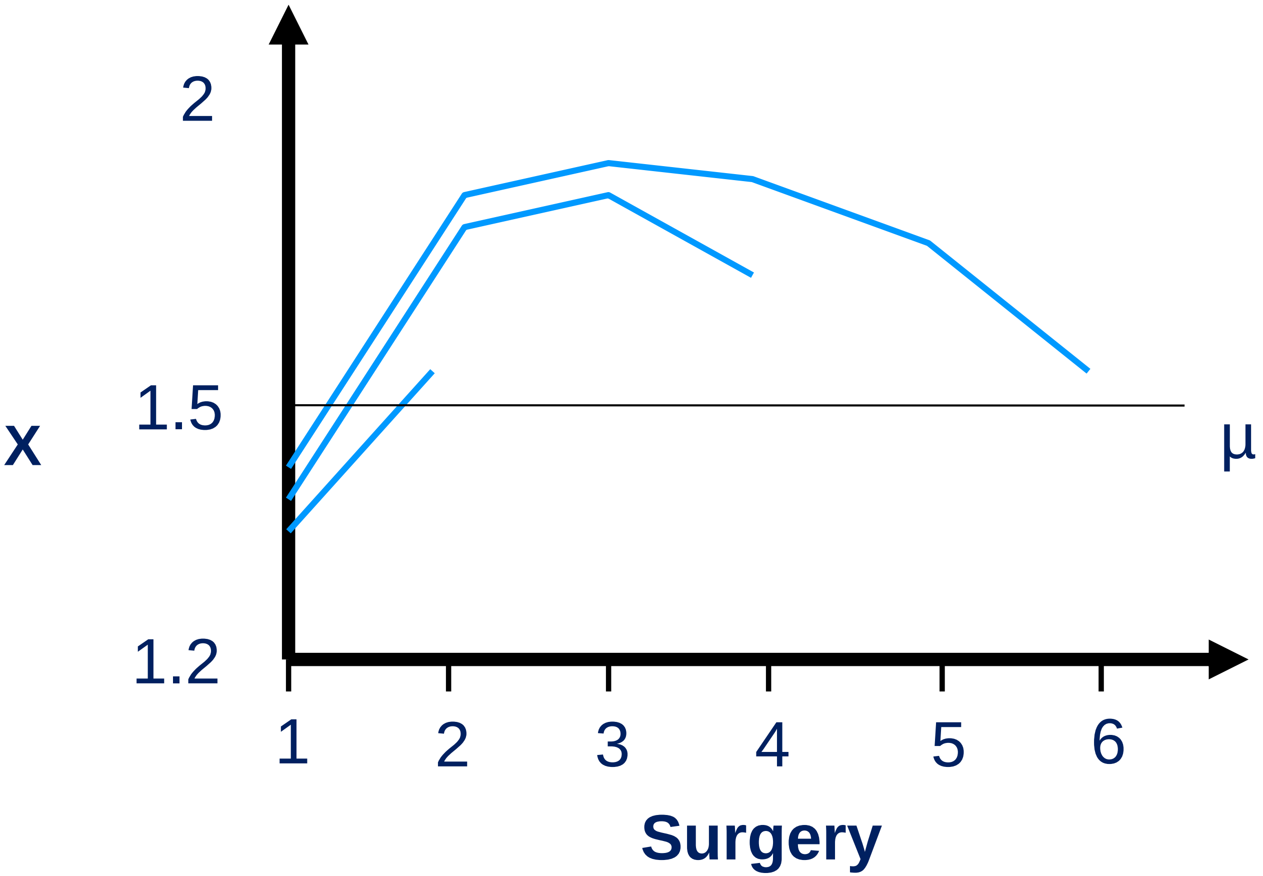
$$\min\{\, Q(\mathbf{x}) = E_{\mathrm{Z}}[Q(\mathbf{x},\mathbf{Z})]\}$$

$$Q(\mathbf{x},\mathbf{Z}^k) = \min\{\, \mathbf{c}\cdot\mathbf{y}^k \mid \underbrace{T\,\mathbf{x} + W\,\mathbf{y}^k}_{\begin{pmatrix} \mathrm{T} & \mathrm{W}^1 & & & \\ \mathrm{T} & & \mathrm{W}^2 & & \\ \mathrm{T} & & & \mathrm{W}^3 & \\ \vdots & & & & \ddots \\ \mathrm{T} & & & & \mathrm{W}^{\mathrm{K}} \end{pmatrix}} = \mathbf{h}^k, \mathbf{y}^k \ge 0\}$$

Solve using L-shaped method



Example: Surgery allocations for n=3, 5, 7 patients with i.i.d. U(1,2)



Insights

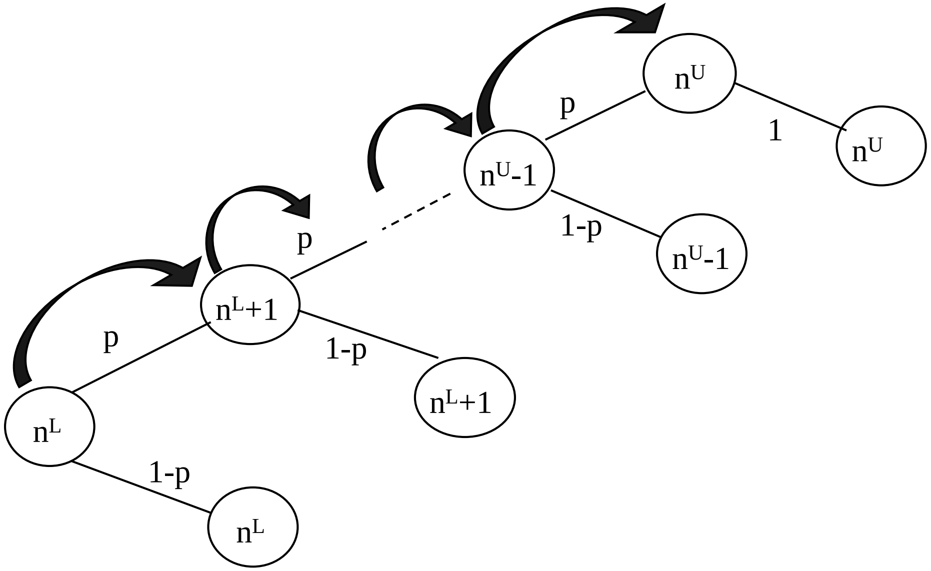
- Simple heuristics often perform poorly
- The value of the stochastic solution (VSS) can be high
- Large instances of this problem can be solved very easily

1) Denton, B.T., Gupta, D., 2003, A Sequential Bounding Approach for Optimal Appointment Scheduling, *IIE Transactions*, 35, 1003-1016

2) Denton, B.T., Viapiano, J, Vogl, A., 2007, Optimization of Surgery Seqencing and Scheduling Decisions Under Uncertainty, *Health Care Management Science,* 10(1), 13-24

There are many variations on this problem

- No-shows
- Tardy arrivals
- Dynamic scheduling
- Robust formulations
- Endogenous uncertainty



Erdogan, S.A., Denton, B.T., “Dynamic Appointment Scheduling with Uncertain Demand,” INFORMS Journal on Computing 25(1), 116-132, 2013.

Erdogan, A, Denton, B.T., Gose, “On-line Appointment Sequencing and Scheduling,” IIE Transactions, 47, 1267-1286, 2015.

Example 2

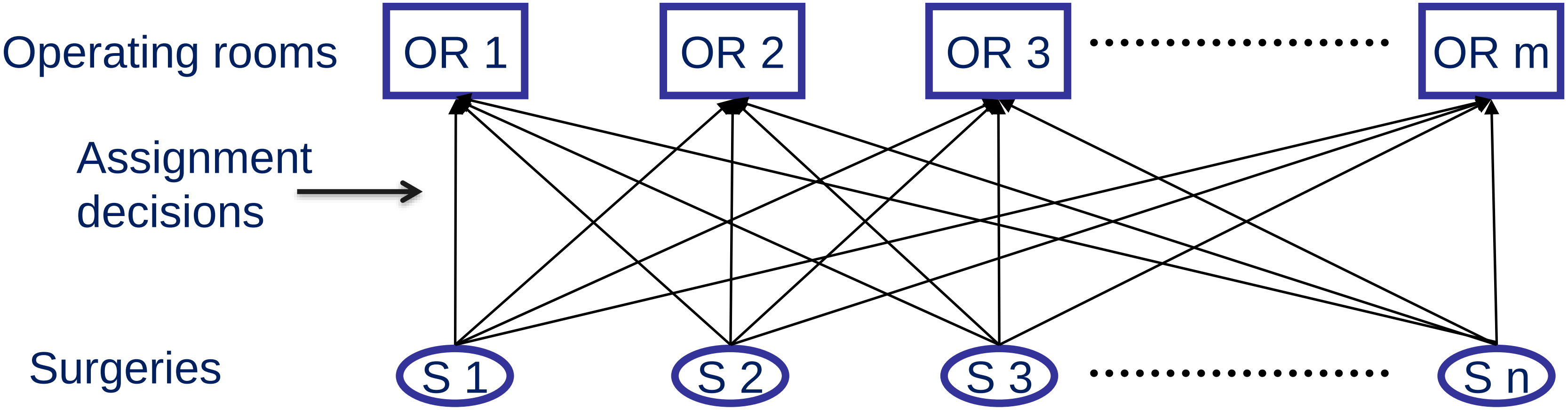
Multiple Operating Room Surgery Allocation

Multi-OR Scheduling Problem

Given a set of surgeries to be scheduled on a certain day decide the following:

- How many ORs to make available to complete all surgeries
- Which OR in which to perform each surgery block

Multi-OR Scheduling Problem



Operating rooms

OR 1

OR 2

OR 3

……

OR m

Assignment decisions

Surgeries

S 1

S 2

S 3

……

S n

Decisions:

- How many ORs to open each day?
- Which OR to schedule each surgery block in?

Extensible Bin-Packing

$$x_i = \begin{cases} 1 \;\; if\; OR\; i\; active \\ 0 \;\; otherwise \end{cases} \qquad y_{ij} = \begin{cases} 1 \;\; if\; surgery\;\; j\; assigned\; to\; OR\; i \\ 0 \;\;\; Otherwise \end{cases}$$

$$Z = \min\{\sum_{i=1}^{m} c^f x_i + c^v o_i\}$$

← Cost of ORs + Overtime

$$s.t. \qquad y_{ij} \le x_i \quad i = 1,\ldots,m,\; j = 1,\ldots,n$$

← Surgeries only scheduled in ORs that are active

$$\sum_{i=1}^{m} y_{ij} = 1 \quad j = 1,\ldots,n$$

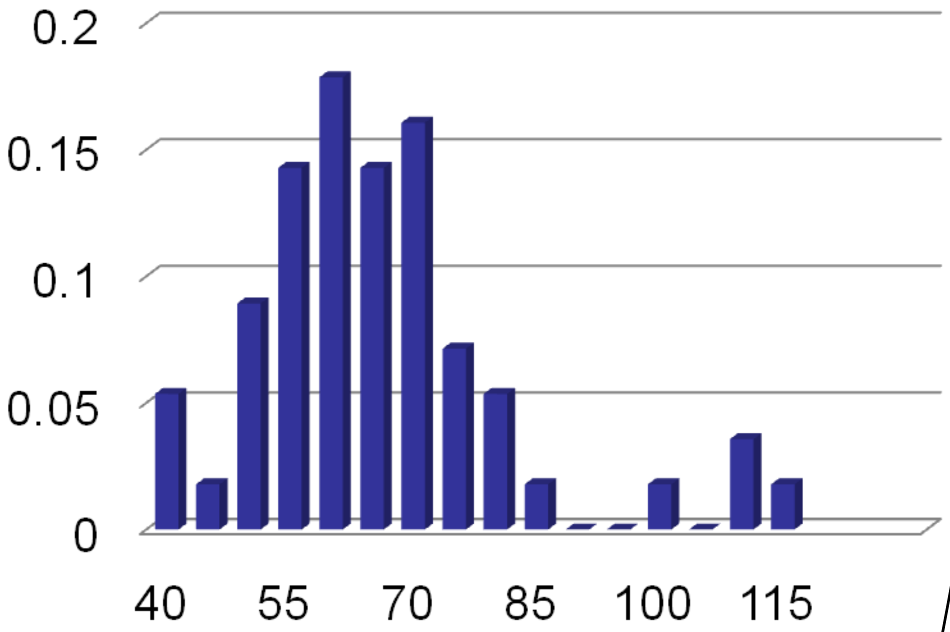
← Every surgery goes in one OR

$$\sum_{j=1}^{n} d_j y_{ij} - o_i \le T x_i \quad i = 1,\ldots,m$$

← Overtime if surgery goes past end of day of length T

$$y_{ij}, x_i\; binary, \quad o_i \ge 0$$

Stochastic MIP with random surgery durations



$$Q(\mathbf{x}) = \min\{\sum_{j=1}^{m} c^{f} x_j + c^{v} E_{\omega}[o_j(\omega)]\}$$

$$s.t. \quad y_{ij} \le x_j \quad \forall(i,j)$$

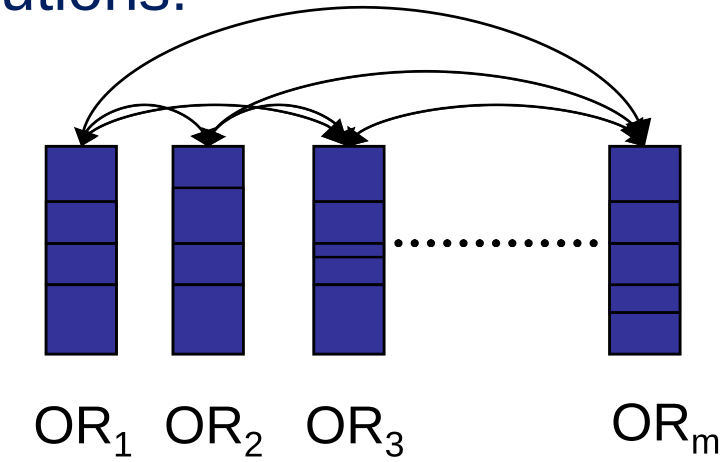
$$\sum_{j=1}^{m} y_{ij} = 1 \;\; \forall(i)$$

$$\sum_{i=1}^{n} d_i(\omega) y_{ij} - o_j(\omega) \le T x_j \quad \forall(i,j,\omega)$$

$$y_{ij}, x_j \in \{0,1\}, \quad o_j(\omega) \ge 0, \forall \omega$$

Symmetry is a problem

There are m! optimal solutions:



Adding the following anti-symmetry constraints reduces computation time:

$$x_1 \geq x_2$$
$$x_2 \geq x_3$$
$$\vdots$$
$$x_m \geq x_{m-1}$$

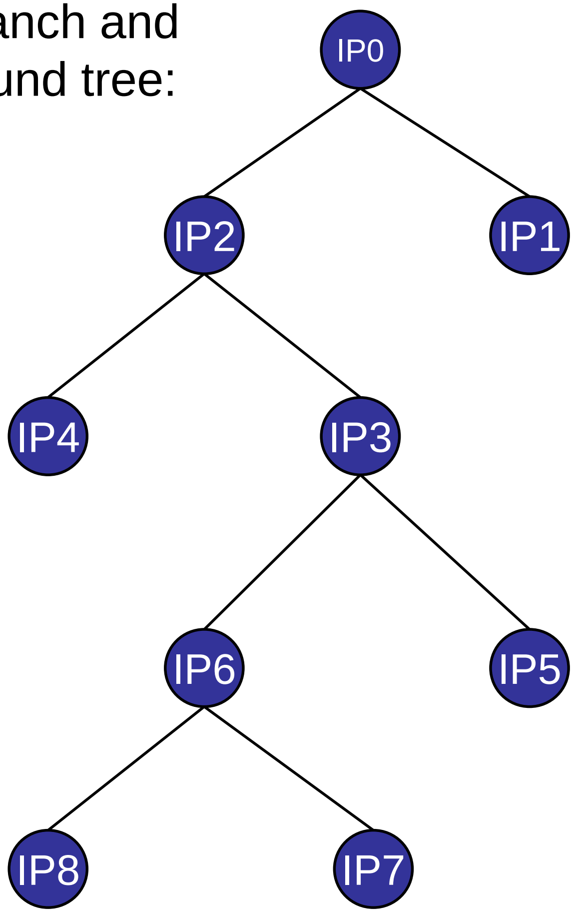
OR Ordering

$$y_{11} = 1$$
$$y_{21} + y_{22} = 1$$
$$\vdots$$
$$\sum_{j=1}^{m} y_{mj} = 1$$

Surgery Assignment

Integer L-Shaped Method

Branch and bound tree:



Master Problem:

$$Z = \min\{ \sum_{j=1}^{m} c^{f} x_{j} + \Theta \}$$

$$s.t. \quad y_{ij} \le x_{j} \quad \forall (i, j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \ \ \forall (i)$$

$$y_{ij}, x_{j} \in \{0,1\}, \Theta \ge 0$$

(optimality cuts)

$$\Theta \ge E_{\omega}[\pi(h - Tx)]$$

Longest Processing Time First Heuristic

```
Sort surgeries in LPT order;
$m \leftarrow$ LB on number of ORs;
while($o_j = 0, \forall j$)
        LPT(m);
        $m \leftarrow m + 1$;
end
Compute $m^*$ with lowest total cost
```

Dell'Ollmo, Kellerer, Speranza, Tuza, *Information Processing Letters* (1998) – provides a 13/12 approximation algorithm for bin packing with extensible bins

Robust Formulation

Robust formulation seeks to minimize the worst case cost.

$$Z = \min\{\sum_{j=1}^{m} c^{f} x_j + F(x,y)\}$$

$$s.t. \quad y_{ij} \leq x_j \quad \forall (i,j)$$

$$\sum_{j=1}^{m} y_{ij} = 1 \quad \forall (i)$$

$$y_{ij}, x_j \in \{0,1\} \geq 0$$

Worst case (adversary) problem

$$F(x,y) = \left\{ \begin{array}{l} \max\limits_{\delta}\{\sum_{j=1}^{m} \eta_j\} \\ s.t. \quad \eta_j = c_j^{v} \max\{0, \sum_{i:y_{ij}=1} \delta_{ij} y_{ij} - d x_j\}, \quad \forall j \\ \sum_{(i,j):y_{ij}=1}^{m} \frac{\delta_{ij} - \underline{z}_i}{\bar{z}_i - \underline{z}_i} y_{ij} \leq \tau \quad \longleftarrow \text{Uncertainty budget} \\ \underline{z}_i \leq \delta_{ij} \leq \bar{z}_i, \forall (i,j) : y_{ij} = 1 \end{array} \right.$$

Results of sample test problems

Instance	1	2	3	4	5	6	7	8	9	10	Avg
LPT	.82	.97	.85	.93	.95	.85	.94	.97	.97	.92	.92
MV	.81	.95	.85	.92	.90	.86	.93	.89	.96	.86	.90
Robust	.93	.97	.97	.92	.89	.94	.92	.90	.97	.92	.92

Table 1: Cost of 0.5 hours overtime equal cost, c^f, of opening an OR

Instance	1	2	3	4	5	6	7	8	9	10	Avg
LPT	1.0	1.0	1.0	1.0	1.0	.99	.99	.97	.99	1.0	.99
MV	1.0	1.0	1.0	1.0	.99	.99	.97	.97	.98	1.0	.99
Robust	.95	1.0	.95	.93	.94	.88	.97	.99	.96	.90	.95

Table 2: Cost of 2 hours overtime equal cost, c^f, of opening an OR

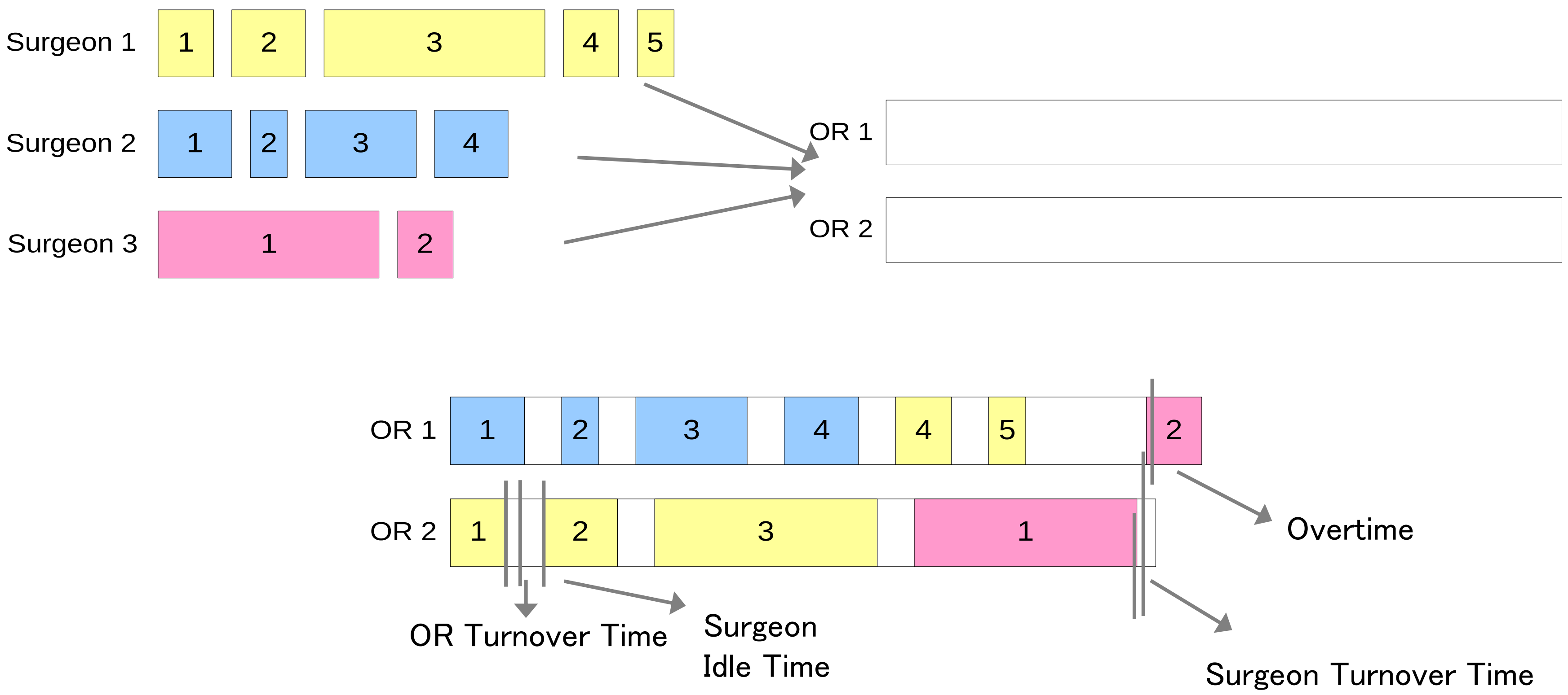
LPT = longest processing time first heuristic, MV = mean value problem, Robust = solution to robust integer program. Results expressed as the ratio of optimal solution to solution generated by MV, LPT, Robust

Insights

- LPT works well when overtime costs are low
- LPT is better (and much easier) than solving MV problem in most cases
- Robust IP is better than LPT when overtime costs are high

Denton, B.T., Miller, A., Balasubramanian, H., Huschka, T., 2010, Optimal Surgery Block Allocation Under Uncertainty, *Operations Research* 58(4), 802-816, 2010

Relaxing assumptions about assignment decisions leads to challenging problems



Batun, S., Denton, B.T., Huschka, T.R., Schaefer, A.J., The Benefit of Pooling Operating Rooms Under Uncertainty, *INFORMS Journal on Computing*, 23(2), 220-237, 2012.

LPT Heuristic Analysis

Extension to Dell'Ollmo et al. (1998) to consider extensible bins with costs

Theorem: The LPT heuristic has the following *performance ratio:*

$$\frac{C^{LPT}}{C^*} \leq \frac{Sc^v}{12c^f}$$

and there exist instances where the bound is tight.

Bam, M., Denton, B.T., Van Oyen, M.P, Cowen, M.E., Surgery Scheduling with Recovery Resources, *IIE Transactions*, 2017 (in press)

Berg, B.P., Denton, B.T., Fast Approximation Methods for Online Scheduling of Outpatient Procedure Centers, *INFORMS JOC*, 2017 (in press)

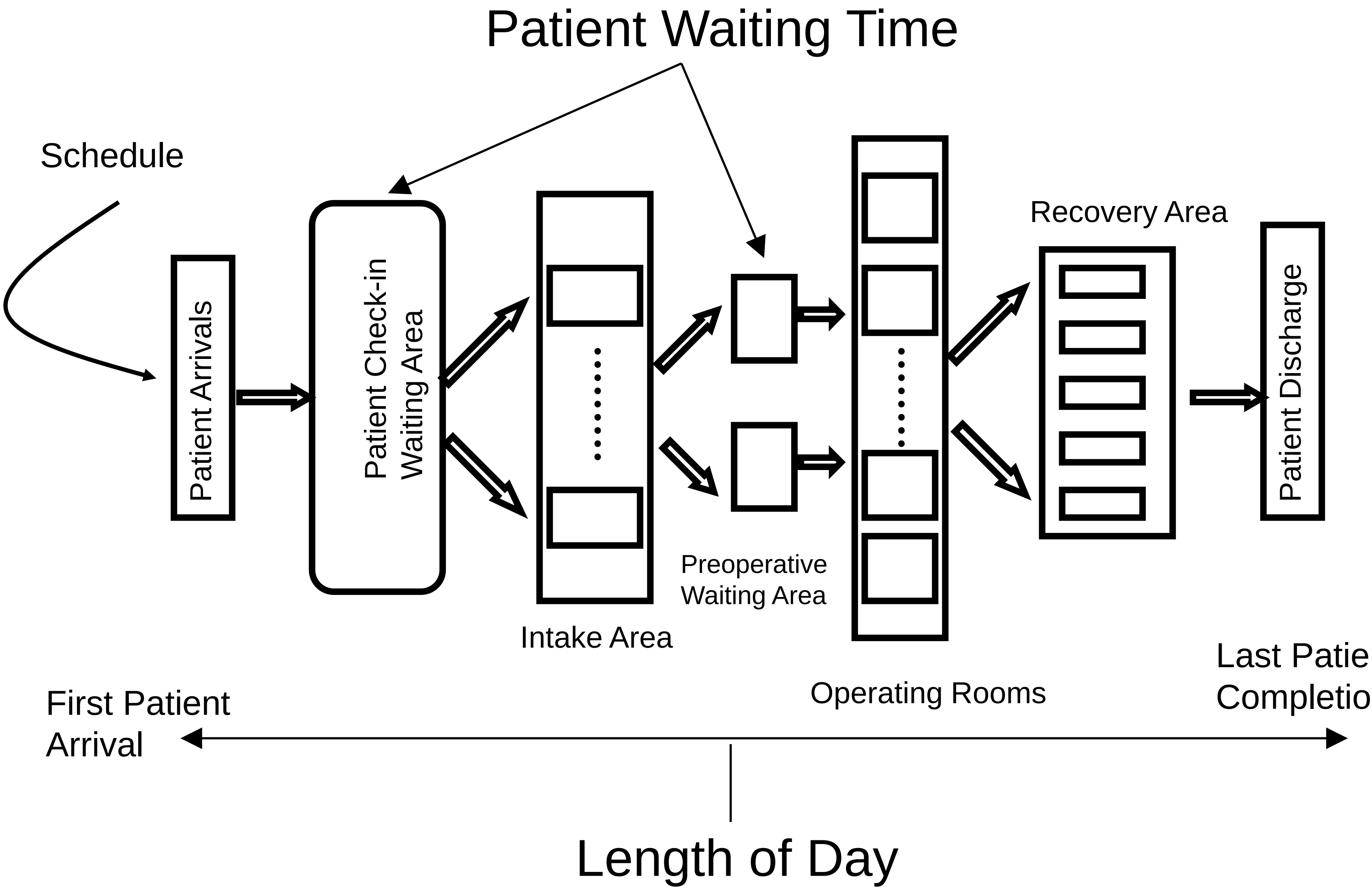
Example 3
Patient Arrival Scheduling in Multi-Stage Procedure Center

Patient Arrival Scheduling Problem

Find the Pareto optimal appointment times for patients having a procedure in an ambulatory surgery center to trade-off:

- Expected patient waiting
- Expected length of day

Endoscopy Suite



Patient Waiting Time

Schedule

Patient Arrivals

Patient Check-in Waiting Area

Intake Area

Preoperative Waiting Area

Operating Rooms

Recovery Area

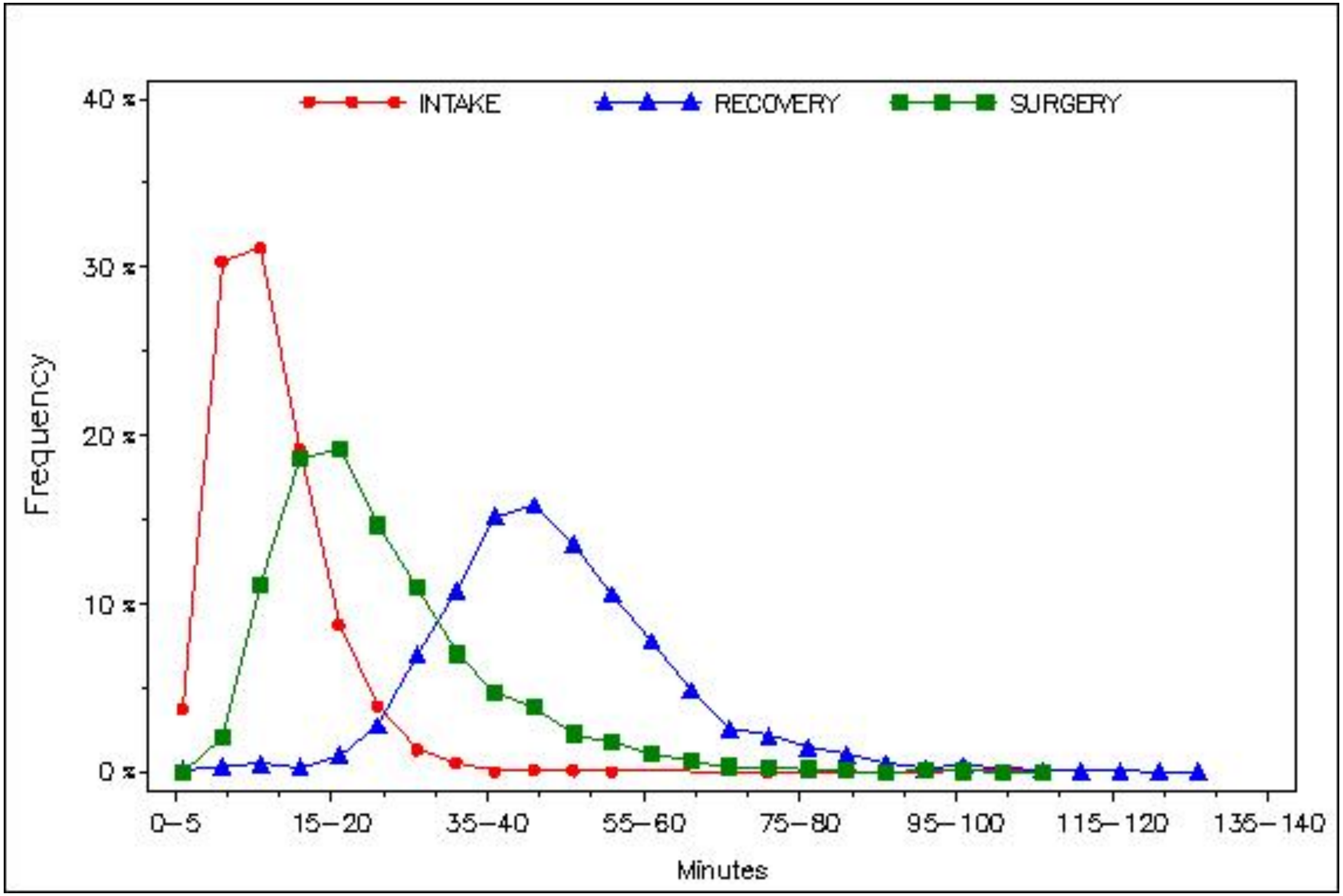
Patient Discharge

First Patient Arrival

Last Patient Completion

Length of Day

Intake, Procedure and Recovery Distributions



Simulation-optimization

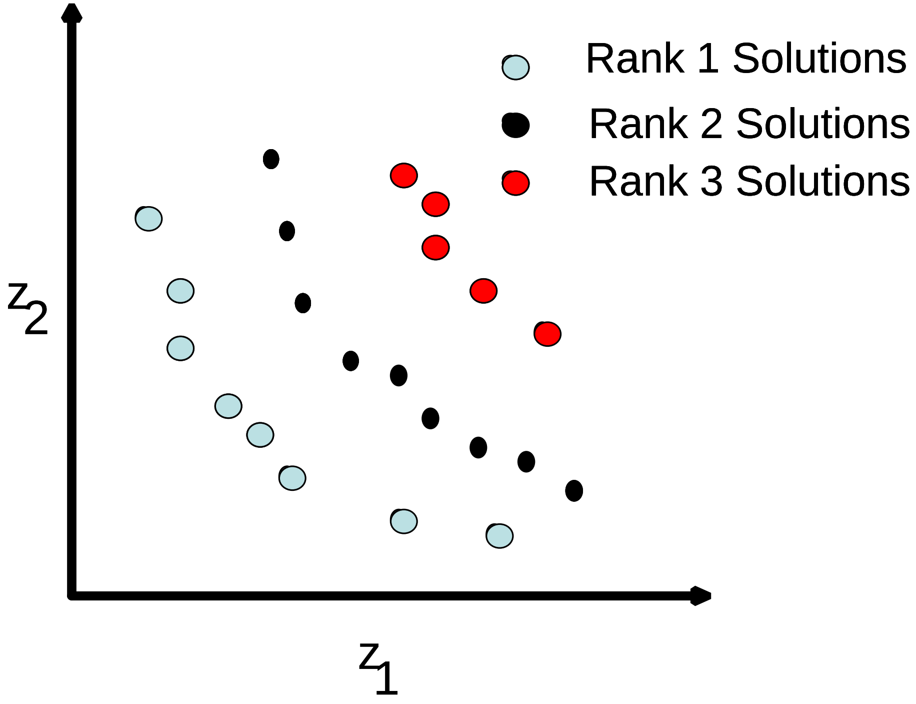
Decision variables: scheduled start times to be assigned to *n* patients each day

Goal: Generate Pareto optimal schedules to understand tradeoffs between patient waiting and length of day

- Schedules generated using a genetic algorithm (GA)
- Non-dominated sorting used to identify the Pareto set and feedback into GA

Pareto Set

The non-dominated sorting genetic algorithm (NSGA-II) of Deb *et al.*(2000):



Selection Procedure

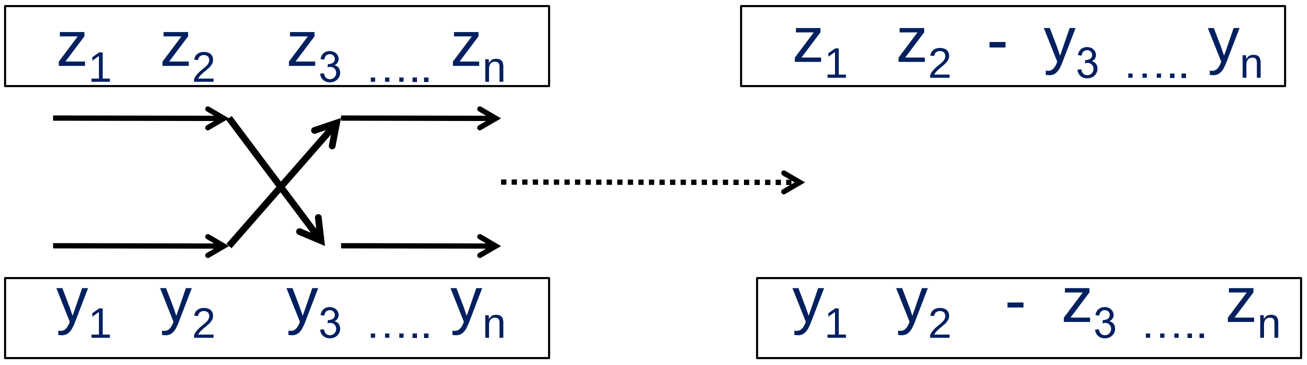
Sequential two stage indifference zone ranking and selection procedure of Rinott (1978) to compute the number of samples necessary to determine whether a solution *i* “dominates” *j*

Solution *i* “dominates” *j* if:

$$E[W_i] < E[W_j] \quad \text{and} \quad E[L_i] < E[L_j]$$

Genetic Algorithm

- Randomly generated initial population of schedules
- Selection based on 1) ranks and 2) crowding distance
- Mutation
- Single point crossover:



z_1 z_2 z_3 ….. z_n

y_1 y_2 y_3 ….. y_n

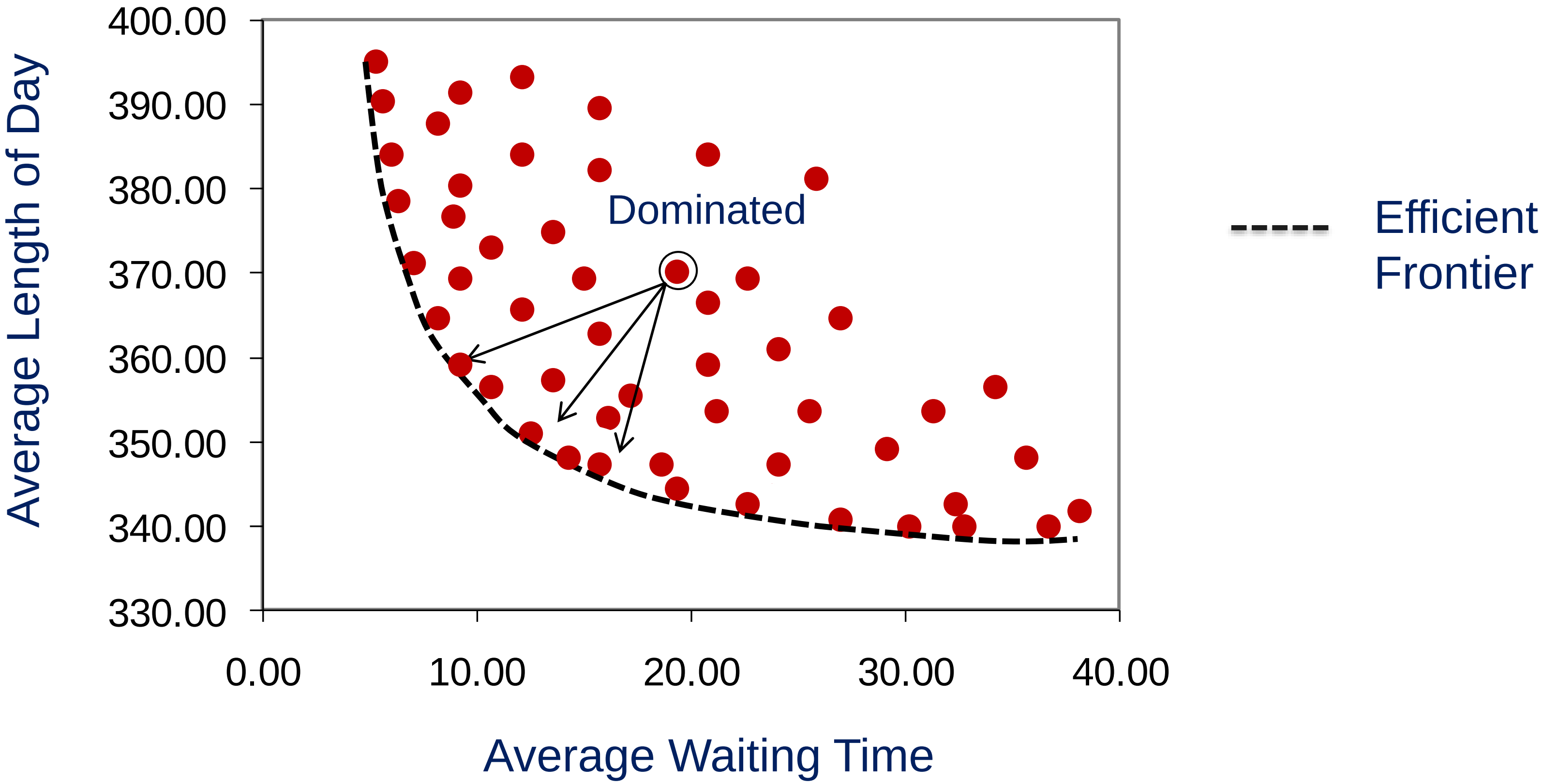
z_1 z_2 - y_3 …. y_n

y_1 y_2 - z_3 ….. z_n

Parents

Children

Schedule Optimization

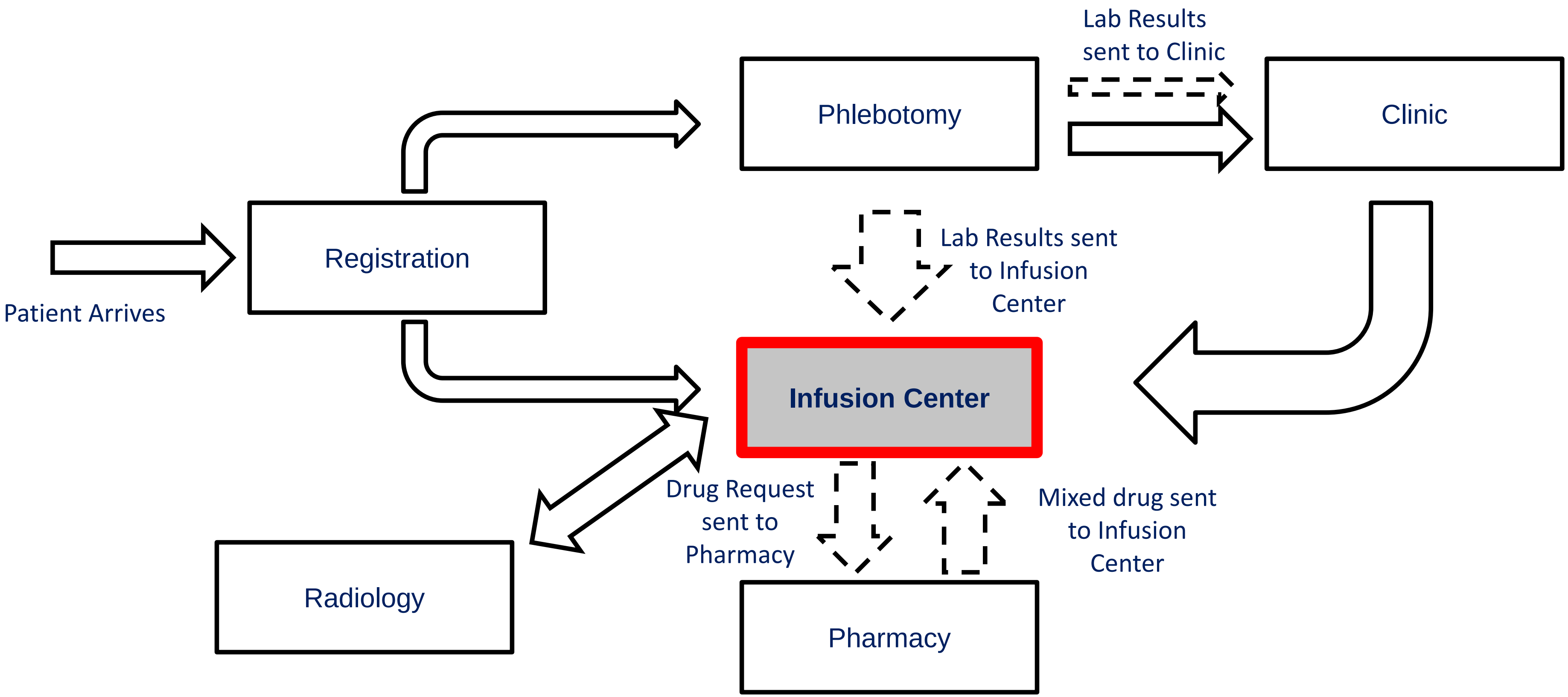


Insights

- A simple simulation optimization approach provides significant improvement to schedules used in practice
- Controlling the mix of surgeries each day can improve both patient waiting time and overtime

Gul, S., Denton, B.T., Fowler, J., 2011 Bi-Criteria Scheduling of Surgical Services for an Outpatient Procedure Center, *Production and Operations Management*, 20(3), 406-417

Many healthcare delivery systems have complex interactions



Lab Results sent to Clinic

Phlebotomy

Clinic

Registration

Lab Results sent to Infusion Center

Patient Arrives

Infusion Center

Drug Request sent to Pharmacy

Mixed drug sent to Infusion Center

Radiology

Pharmacy

Woodall, Jonathan C., Tracy Gosselin, Amy Boswell, Michael Murr, and Brian T. Denton. "Improving patient access to chemotherapy treatment at Duke Cancer Institute." *Interfaces* 43, no. 5 (2013): 449-461.

Key Points

- There are many open opportunities for research in optimization of healthcare delivery systems

- New problems help drive creation of new methods and theory

Acknowledgements

Hari Balasubramanian (University of Massachusetts)
Sakine Batun (University of Pittsburgh)
Maya Bam (University of Michigan)
Bjorn Berg (George Mason University)
Ayca Erdogan (San Jose State University)
Serhat Gul (TED University)
Todd Huschka (Mayo Clinic)
Andrew Miller (University of Bordeaux)
Heidi Nelson (Mayo Clinic)
Andrew Schaefer (Rice University)
Zheng Zhang (University of Michigan)

Some of this work was funded in part by grants from the *Service Enterprise Systems* program at the *National Science Foundation*

Thank You

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