# IO543: Scheduling

## Lecture 12

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# Today

- More about $Fm\,|block|C_{max}$
- Assembly Lines
- Midterm Exam

# $Fm\,|block|C_{max}$ and the TSP

- For $F2\,|block|C_{max}$, whenever a job starts on machine 1 the preceding job starts on machine 2

- The time job $J_k$ spends on machine 1, in process or blocked, is $\max(p_{1,Jk}, p_{2,Jk-1})$

- $F2\,|block|C_{max}$ can be formulated as a TSP with n+1 "cities" and distance from "city" j to k:

$$d_{jk} = \max(p_{1k}, p_{2j})$$

$$d_{0k} = p_{1k}$$

$$d_{j0} = p_{2j}$$

n jobs plus a dummy job, 0, representing the start "city"

# $Fm\,|block|C_{max}$ Heuristic



## Profile Fitting Heuristic:

**Step 1**: Choose the job, $j^*$, with lowest total processing time. Compute its departure time from each machine:

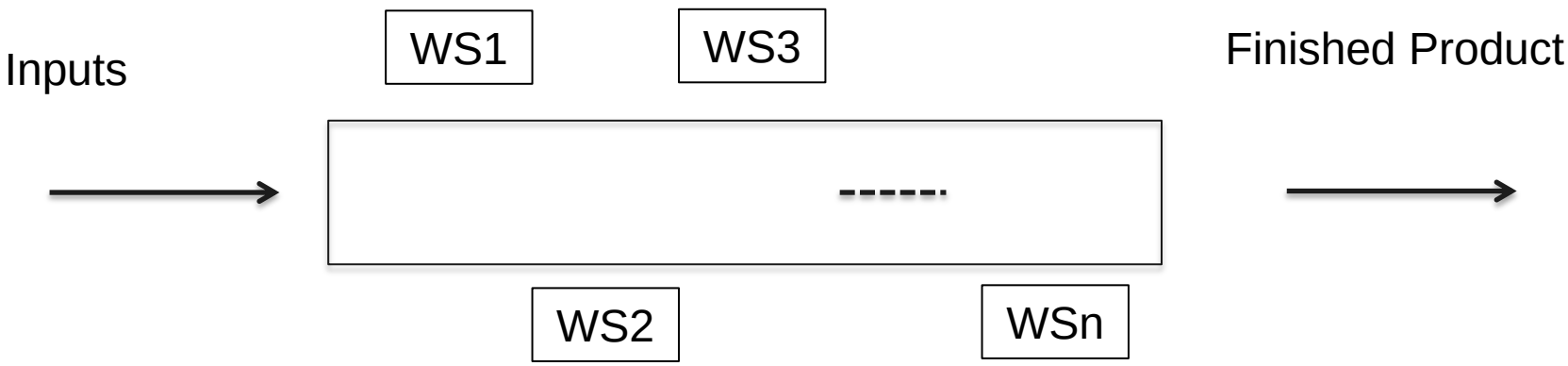
$$D_{i,j^*} = \sum_{h=1}^{i} p_{h,j^*}$$

**Step 2**: For each pending job compute the total wasted time (idle and blocked time) on all machines if it is scheduled next.

**Step 3**: Select the job with minimum total wasted time as next in the sequence; remove the job from the list of pending jobs; if the list of pending jobs is empty STOP, otherwise go to Step 2.

# Assembly Lines

A simple **assembly line** is a special case of a **flow shop** in which material moves continuously at a **uniform rate** through a sequence of workstations



# Example

Identical products are processed on a series of 5 machines. Each product has the same processing time on a particular machine.

What do you expect to happen?

| Machine /Jobs | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| $p_{1,Jk}$ | 1 | 1 | 1 | 1 | 1 |
| $p_{2,Jk}$ | 2 | 2 | 2 | 2 | 2 |
| $p_{3,Jk}$ | 3 | 3 | 3 | 3 | 3 |
| $p_{4,Jk}$ | 4 | 4 | 4 | 4 | 4 |

# Example

Identical products are processed on a series of 5 machines. Each product has the same processing time on a particular machine.

What do you expect to happen?

| Machine /Jobs | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| $p_{1,Jk}$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| $p_{2,Jk}$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| $p_{3,Jk}$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |
| $p_{4,Jk}$ | 2.5 | 2.5 | 2.5 | 2.5 | 2.5 |

# Example

Identical products are processed on a series of 5 machines. Each product has the same processing time on a particular machine.

What do you expect to happen?

| Machine /Jobs | J1 | J2 | J3 | J4 | J5 |
|---|---|---|---|---|---|
| $p_{1,Jk}$ | 1 | 1 | 1 | 1 | 1 |
| $p_{2,Jk}$ | 6 | 6 | 6 | 6 | 6 |
| $p_{3,Jk}$ | 3 | 3 | 3 | 3 | 3 |
| $p_{4,Jk}$ | 2 | 2 | 2 | 2 | 2 |

# Assembly Line Balancing

- **Assembly line balancing** involves distributing workload across workstations to reduce **idle time**, smooth **workload**, and/or minimize **makespan**

- Learn more about line balancing with the following video:

http://www.youtube.com/watch?v=zLY-Couxmhk

# Reminder

**Course Grading:**

Homework Assignments: 15%

Midterm: 30%

In-Class Participation: 15%

Final Exam (in class, closed book): 40%

# Statistics



Statistics on homework and in-class assignments:

| | Mean | Stdev |
|---|---|---|
| Homework 1 (15pts) | 84% | 14% |
| Homework 2 (12pts) | 96% | 21% |
| Homework 3 (18pts) | 90% | 10% |
| In Class Assignments (3pts) | 93% | 21% |

Solutions for assignment 4 posted by Saturday.

# Rules About the Midterm

- Available on ctools at 5pm on Feb 22
- Return no later than start of class Feb 25
- You may use Pinedo Textbook, IOE543 lecture slides and notes, assignment solutions, AMPL documentation. You may not use any other resources or communicate with anyone about the midterm.
- You can use AMPL if called for but you must provide all of the details about how you obtained solutions including a detailed written description of the IP formulation, AMPL code and output, and an explanation of the results.

# Structure of the Midterm

Some short answer questions such as:

- True or False…
- Why does the Slope Heuristic tend to work well?
- Explain why $P2||Cmax$ is NP Hard

Questions similar to assignment questions:

- At least one question requiring computation using AMPL
- At least one theory question

# Textbook Reading

## Important readings from Pinedo to augment the lectures:

| Topic | Readings |
|---|---|
| Introduction to Scheduling | Chapters 1 |
| Introduction to Complexity and Single Server Scheduling | Chapter 2 |
| Single Server Scheduling | Chapter 3 |
| Advanced Single Server Scheduling | Chapter 4: 4.1, 4.2, 4.3 (we covered $1\,\vert s_{jk} \vert C_{max}$ but not at the full level of detail discussed in 4.4) |
| Parallel Server Scheduling | Chapter 5: 5.1, 5.2, 5.3 |
| Flow shops | Chapter 6: 6.1, 6.2 |

# Tips on Integer Programming

When formulating integer programs:

1. Define decision variables
2. Define the objective function; if not possible go back to Step 1.
3. Define the constraints; if not possible go back to Step 1

Note: watch out for <u>nonlinearity</u>

# Review of Methods

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## How do you find optimal schedules for the following problems?

- $1 \,|\,| L_{max}$ → EDD
- $1 \,|r_j\,| L_{max}$ → B&B
- $1 \,|\,|\, \sum_j C_j$ → SPT
- $1 \,|\,|\, \sum_j w_j C_j$ → WSPT
- $1|prec|\sum_j w_j C_j$ → WSPT, rho factors
- $1 \,|\,|\, \sum_j U_j$ → Knapsack IP
- $1 \,|\,|\, \sum_j T_j$ → DP
- $1 \,|\,|\, \sum_j w_j T_j$ → IP
- $1 \,|s_{jk}\,|\, C_{max}$ → TSP IP
- $Pm \,|\,| C_{max}$ → IP
- $Pm \,|prmp\,| C_{max}$ → Wrap around
- $\mathrm{P}m \,|\,|\, \sum_j C_j$ → SPT
- $\mathrm{Pm} \,|\,|\, \sum_j w_j C_j$ → IP
- $\mathrm{P}\infty \,|\,| C_{max}$ → Crit. Path Method
- $F2 \,|prmu|\, C_{max}$ → Johnson
- $F2 \,|block|\, C_{max}$ → TSP
- $Fm \,|\,|\, C_{max}$ → IP