Lecture 19 Example (EVPI and VSS for Scheduling Move Start Times)

<u>Problem:</u> The Michigan Theatre schedules its first evening movie at 7pm. The total time for the showing, clearing theater, and setup for the next movie is exponentially distributed with a mean of 100 minutes. The manager estimates a loss of $2/minute for the theater sitting idle between movies, and a loss of $5/minute that the second movie starts late.

a) What is the Expected Value of Perfect Information (EVPI)?
b) What is the Value of the Stochastic Solution (VSS)?

Part (a) EVPI Calculation:

$$EVPI = E_\omega[5\max\{0, Z(\omega) - x^*\} + 2\max\{0, x^* - Z(\omega)\}]$$
$$- E_\omega[5\max\{0, Z(\omega) - x(\omega)\} + 2\max\{0, x(\omega) - Z(\omega)\}]$$

Note that for this particular problem, the 3rd and 4th terms in the EVPI expression are zero since $x(\omega) = Z(\omega)$. Therefore:

$$EVPI = E_\omega[5\max\{0, Z(\omega) - x^*\} + 2\max\{0, x^* - Z(\omega)\}]$$

Evaluate the following two integrals for the first two terms of EVPI equation:

Reminder (Integration by parts): $\int udv = uv - \int vdu$, set $u = z$,

Integral 1:

$$\int_0^\infty \max\{0, z - x\}\lambda e^{-\lambda z}dz = \int_x^\infty (z - x)\lambda e^{-\lambda z}dz = \int_x^\infty \lambda z e^{-\lambda z}dz - \int_x^\infty \lambda x e^{-\lambda z}dz$$

$$\Longrightarrow = -ze^{-\lambda z}|_x^\infty + \int_x^\infty e^{-\lambda z}dz + xe^{-\lambda z}|_x^\infty$$
$$= xe^{-\lambda x} + \frac{1}{\lambda}e^{-\lambda x} - xe^{-\lambda x}$$
$$= \frac{1}{\lambda}e^{-\lambda x}$$

Integral 2:

$$\int_0^x \max\{0, x-z\}\lambda e^{-\lambda z}dz = \int_0^x (x-z)\lambda e^{-\lambda z}dz = \int_0^x x\lambda e^{-\lambda z}dz - \int_0^x z\lambda e^{-\lambda z}dz$$
$$= x\left(1-e^{-\lambda z}\right) + ze^{-\lambda z}|_0^x - \int_0^x e^{-\lambda z}dz$$
$$= x + \frac{1}{\lambda}e^{-\lambda x} - \frac{1}{\lambda}$$

Using the above results and the fact that $x^* = 125.2$ is the optimal solution to $1\,|appt\,|(c^w\sum w_j + c^s\sum s_j)$, i.e., the optimal time to schedule for the first movie (see lecture 19 slides) we have:

$EVPI = 5\left(100e^{-\frac{125.2}{100}}\right) + 2(125.2 + 100e^{-\frac{125.2}{100}} - 100)$ = 250.60

Thus the value of being able to perfectly predict the time needed for the first movie and scheduling that amount of time for the movie is $250.60.

Part (b): VSS Calculation

The Michigan Theatre **mean value problem** is:

$$\min\{5\max\{0, E_\omega[Z(\omega)] - x\} + 2\max(0, x - E_\omega[Z(\omega)]\}\}$$

Obviously $x^{mv} = E_\omega[Z] = 100$ will minimize the objective. Thus, the equation for VSS is

$$VSS = E_\omega[5\max\{0, Z - x^{mv}\} + 2\max\{0, x^{mv} - Z\}] - E_\omega[5\max\{0, Z - x^*\} + 2\max\{0, x^* - Z\}]$$
$$= 5\left(100e^{-\frac{100}{100}}\right) + 2\left(100 + 100e^{-\frac{100}{100}} - 100\right)$$

= 257.3-250.60

= 6.91

Therefore the value of solving $1\,|appt\,|(c^w\sum w_j + c^s\sum s_j)$ and using the optimal solution results in $6.91 of savings compared to solving (the much easier) mean value problem.