IO543: Scheduling

Lecture 10

Brian Denton
Associate Professor, IOE

IOE Bld 2893
btdenton@umich.edu

Announcements

- Assignment 2 solutions on ctools
- Assignment 4 due Feb 18; Solutions posted by Feb 20 (keep a copy of your assignment)
- Midterm exam:
  - Take home exam (includes some AMPL)
  - Available by 5pm on Monday, Feb 15
  - No class on Tuesday, Feb 16
  - Midterm exam due no later than 10:40 Feb 18

Today

Advanced parallel machine scheduling models:

- Due dates

Flow Shops:

Due Dates: $Pm||L_{max}$

- $Pm||L_{max}$ is a special case of $\mathrm{Pm}||C_{max}$ and therefore it is **NP-Hard**

- If all jobs have the same **due date**, *d*, then $Pm||L_{max}$ can be formulated as follows**:**

$$\min\ L_{max}$$

$$\begin{aligned} s.t.\quad & L_{max} \geq C_i - d, && for\ all\ i \\ & \textstyle\sum_i x_{ij} = 1, && for\ all\ j \\ & C_i = \textstyle\sum_j p_j x_{ij}\ , && for\ all\ i \\ & L_{max} \geq 0, x_{ij}\ binary, && for\ all\ i,j \end{aligned}$$

Due Dates: $Pm \mid \mid L_{max}$

If **due dates** are **job dependent** the formulation is:

$$\min\ L_{max}$$

$$
\begin{aligned}
s.t.\quad & L_{max} \geq C_{ij}^{k} - d_j, && for\ all\ i,j,k, && \text{(A)}\\
& C_{ij}^{k} \leq M x_{ij}^{k}, && for\ all\ i,j,k, && \text{(B)}\\
& C_{ij}^{k} \geq \textstyle\sum_{k'=1}^{k} \sum_{j'} p_{j'} x_{ij'}^{k'} - M\left(1 - x_{ij}^{k}\right), && for\ all\ i,j,k, && \text{(C)}\\
& \textstyle\sum_{k} \ \sum_{i} x_{ij}^{k} = 1, && for\ all\ j\ , && \text{(D)}\\
& \textstyle\sum_{j} x_{ij}^{k} \leq 1, && for\ all\ i,k, && \text{(E)}\\
& L_{max} \geq 0, C_{ij}^{k}, \geq 0, x_{ij}\ binary, && for\ all\ i,j,k &&
\end{aligned}
$$

New Topic

Flow Shops

Examples

http://www.youtube.com/watch?v=jLud5XYfY_c

Flow Shops

There are many types of flow shops:

- Flow shops with unlimited intermediate storage: $Fm \mid \mid \gamma$
- Flow shops with limited intermediate storage: $Fm \mid block \mid \gamma$
- Flexible flow shops: $FFc \mid \mid \gamma$

$Fm \mid\mid C_{max}$

- Flow shops with unlimited storage arise when storage requirements are minimal:
  - Integrated circuits
  - Printed circuit boards
  - Smartphones
- For large items there is a limit on buffer space:
  - Cars
  - Photocopiers
  - Refrigerators

Flow Shops

Makespan is a common criteria for flow shops:

- Consistent with maximizing machine utilization
- Easier to solve than total completion time and due date related objectives
- We will focus on: $Fm||C_{max}$, $Fm|block|C_{max}$, $FFc||C_{max}$

$F2 \mid\mid C_{max}$: Johnson's Rule

An optimal sequence for $F2 \mid\mid C_{max}$ can be obtained by **Johnson's Rule**:

1) Partition the jobs into two sets:

   i) Set I with jobs such that $p_{1j} < p_{2j}$
   ii) Set II with $p_{1j} > p_{2j}$
   iii) Jobs with $p_{1j} = p_{2j}$ can go in either set

2) Set I jobs go first in SPT order (based on p_{1j}'s)

3) Set II jobs go second in LPT order (based on p_{2j}'s)

Johnson's Rule Example

Find **all optimal sequences** for the following instance of $F2 \mid \mid C_{max}$

Jobs	J1	J2	J3	J4	J5
$p_{1,Jk}$	5	5	3	6	3
$p_{2,Jk}$	4	7	3	4	4

Proof of Optimality:

Proof by **contradiction**. Suppose another type of schedule is optimal. Then there is a pair of adjacent jobs (j,k) that satisfy one of three conditions:

1) Job j belongs to Set II and job k to Set I

2) Jobs j and k belong to Set I and $p_{1j} > p_{1k}$

3) Jobs j and k belong to Set II and $p_{2j} < p_{2k}$

Show that under any of these conditions a pairwise interchange reduces the makespan.