IO543: Scheduling

Lecture 17

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Reminder: Mini Project

In groups of 3-4 pick a scheduling application you are interested in and present to the class

Presentations will be in class April 9 and 14.

Due Thursday, March 24: Names of people on your team, team name, a summary (max one page) describing what you plan to cover in your presentation.

Presentation Guidelines

- Presentations should be about 10 minutes in duration
- Everyone in the group should be involved in the presentation
- The presentation should describe the topic and its relationship to topics covered in this class
- Class survey will define the grade which will count for 1/3 of participation component (5% overall); Survey question: “This presentation was clear, interesting, and informative: 1, 2, 3, 4, 5”.
- Resources for presentations:

http://www.nature.com/scitable/ebooks/english-communication-for-scientists-14053993/giving-oral-presentations-14239332
http://www.pcworld.idg.com.au/slideshow/366369/world-worst-powerpoint-presentations/

Sports Applications

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OR PRACTICE

SCHEDULING A MAJOR COLLEGE BASKETBALL CONFERENCE

GEORGE L. NEMHAUSER

Georgia Institute of Technology, Atlanta, Georgia

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Carnegie Mellon University, Pittsburgh, Pennsylvania

(Received February 1997; revision received May 1997; accepted July 1997)

The nine universities in the Atlantic Coast Conference (ACC) have a basketball competition in which each school plays home and away games against each other over a nine-week period. The creation of a suitable schedule is a very difficult problem with a myriad of conflicting requirements and preferences. We develop an approach to scheduling problems that uses a combination of integer programming and enumerative techniques. Our approach yields reasonable schedules very quickly and gave a schedule that was accepted by the ACC for play in 1997–1998.

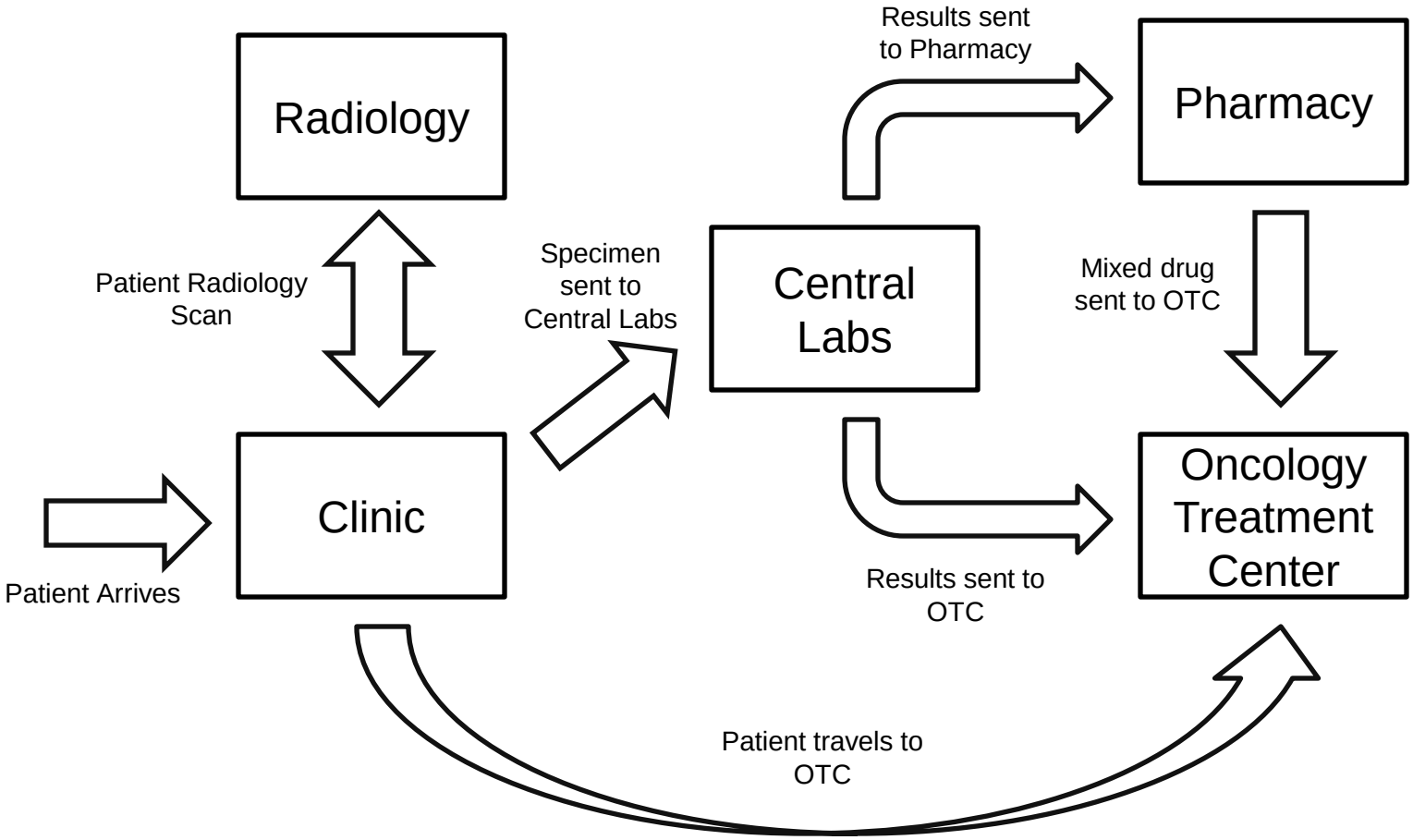
Motivation

In practice many parameters in scheduling models are stochastic:

- Processing Times
- Machine availability (breakdowns)
- Number of jobs
- Release times
- Due dates
- Job routings
- Setup times

Example: Chemotherapy Infusion

Cancer Center Overview



Example: Chemotherapy Infusion

Oncology Treatment Center (OTC):

- Location where chemotherapy received
- Organized in “pods”
- Dependent on many cancer center activities

Example: Chemotherapy Infusion

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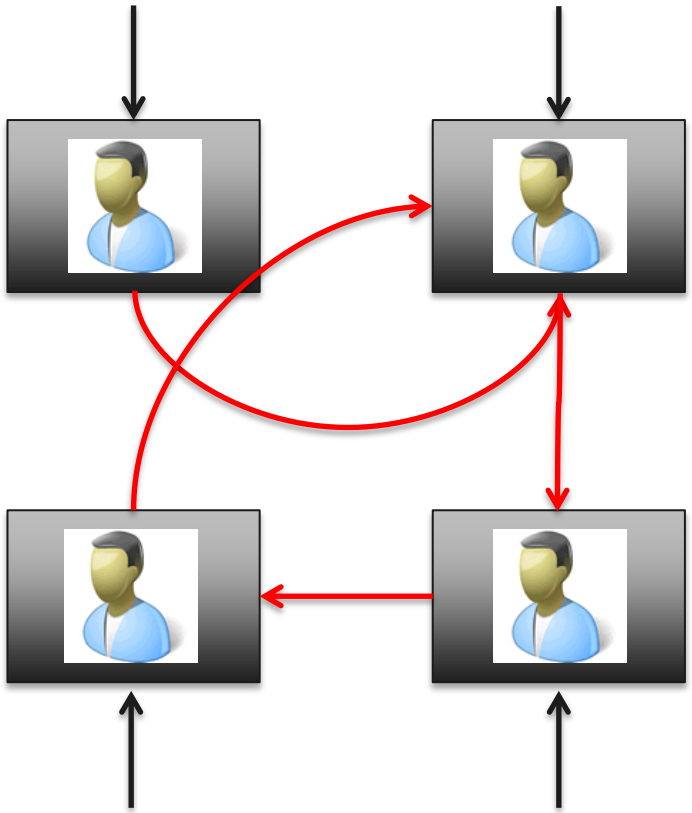
OTC Pod Flow:



Example: Chemotherapy Infusion

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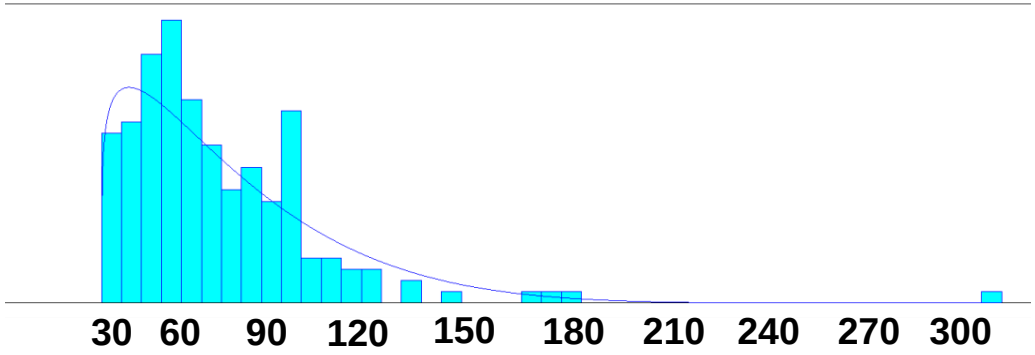
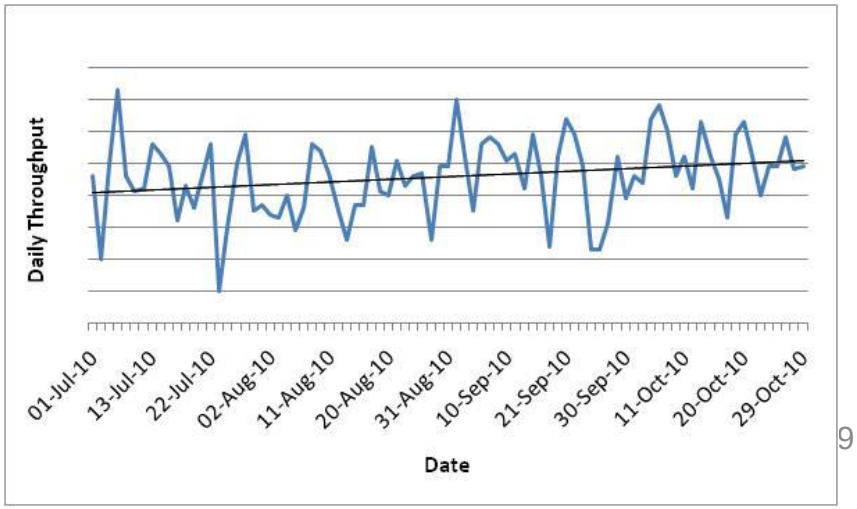
Uncertain scheduling parameters:

- Infusion time
- Patient arrival times
- IV hookup
- Number of patients/day

Infusion Time Distribution



Number of Patients/Day



Learn More (Paper on Ctools):
Woodall et al. "Improving Patient Access to Chemotherapy Treatment at Duke Cancer Institute," Interfaces, 2012

Probability Review

The triplet (Ω, A, P) defines a *probability space*:

- Ω is the set of all outcomes
- A is the set of all subsets of Ω , called *events*
- P is a *probability measure*

Probability spaces must satisfy certain properties:

Axiom 1. $P(\Omega)$ =1
Axiom 2. $0 \leq \mathrm{P(E)} \leq 1$ for any $E \in A$
Axiom 3. For a set of n disjoint events:

$$P\{\cup_{i=1}^{n} A_i\} = \sum_{i=1}^{n} P\{A_i\}$$

Probability Review

A random variable is a function that maps from a probability space to the set of real numbers

<u>Example:</u>

If $\Omega = \{Head, Tail\}$ and $\mathrm{P}\{\mathrm{Head}\} = \mathrm{P}\{\mathrm{Tail}\} = 0.5$ then the function:

$$\mathrm{z}(\omega) = \begin{cases} -1, if\ \omega = "Head" \\ 1, if\ \omega = "Tail" \end{cases}$$

is a random variable.

Types of Probability Distributions

Continuous vs. Discrete Distributions

Discrete distributions have a countable set of possible outcomes

Continuous distributions may assume real values within one or more intervals:

$$F(t) = P\{Z \leq t\} = \int_0^t f(z)dz$$

We refer to $\mathrm{f}(z)$ as the probability density function (pdf) and $F(t)$ as the cumulative distribution function (cdf).

Example: Probability Distribution

The *exponential distribution* is commonly encountered:

$$f(z) = \lambda e^{-\lambda z}$$

The corresponding cdf is:

$$\mathrm{F}(t) = 1 - e^{-\lambda t}$$

The mean of an exponentially distributed random variable, z, is:

$$\mathrm{E[Z]} = \int_0^\infty z f(z) dz = 1/\lambda$$

where λ is the *rate* of the distribution.

Expectations

The expectation of a function of a discrete random variable can be written as

$$E_Z[h(Z)] = \sum_{\omega=1}^{N} p(\omega)h(z(\omega))$$

The expectation of a function of a continuous random variable can be written as

$$E_Z[h(Z)] = \int_0^\infty h(z)f(z)dz$$

Example: $1 \mid \mid \sum C_j$

Consider the following stochastic instance of $1 \mid \mid \sum C_j$. What sequence(s) minimize **expected** total completion time?

Processing Time distributions:

Jobs 1 and 3:

Outcome (ω_j)	1	2
$p(\omega_j)$	1/2	1/2
$t_j(\omega_j)$	1	2

Job 2:

Outcome (ω_2)	1	2
$p(\omega_2)$	2/3	1/3
$t_2(\omega_2)$	2	4

Example: $1 \mid \mid \sum C_j$

What is the **probability distribution** of the total completion time for the following stochastic instance of $1 \mid \mid \sum C_j$ assuming sequence J1→ J2 → J3?

Processing Time Distributions:

Jobs 1 and 3:

Outcome (ω_j)	1	2
$p(\omega_j)$	1/2	1/2
$t_j(\omega_j)$	1	2

Job 2:

Outcome (ω_2)	1	2
$p(\omega_2)$	2/3	1/3
$t_2(\omega_2)$	2	4

More Properties of Random Variables

- Properties of random variables include their **support** (range), **mean, variance**, and other higher order **moments**

- Special properties of random variables can sometimes be used to define an **ordering** of random variables

Definition: A random variable Z_1 is **larger in expectation** than random variable Z_2 if $E_{Z_1}[Z_1] \geq E_{Z_2}[Z_2]$.

Stochastic Dominance

Stochastic orders provide insight beyond the mean of random variables:

Definition: A random variable Z_1 is **stochastically larger** than a random variable Z_2 if:

$$P(Z_1 > t) \geq P(Z_2 > t), \quad for\ all\ t$$

Stochastic Dominance

Definition: If

$$\int_0^\infty h(z_1)f_1(z_1)dz_1 \geq \int_0^\infty h(z_2)f_2(z_2)dz_2$$

for all convex functions $h(\cdot)$ then random variables Z_1 and Z_2 satisfy a **convex ordering.**

This ordering is commonly denoted as $Z_1 \geq_{cx} Z_2$.

Stochastic Dominance

Definition: If

$$\int_0^\infty h(z_1)f_1(z_1)dz_1 \geq \int_0^\infty h(z_2)f_2(z_2)dz_2$$

for all increasing convex functions $h(\cdot)$ then random variables Z_1 and Z_2 have an **increasing convex ordering** denoted as

$$Z_1 \geq_{icx} Z_2.$$

Examples: Stochastic Dominance

Following are some examples of **convex orders**:

- If $X \sim N(\mu, \sigma_X^2)$ and $\mathrm{Y} \sim N(\mu, \sigma_Y^2)$ and $\sigma_X^2 \leq \sigma_Y^2$ then $X \leq_{cx} Y$

- If $X \sim U(a, b)$ and $\mathrm{Y} \sim U(c, d)$ such that $E[X] = E[Y]$ and $Var\ (X) \leq Var(Y)$ then $X \leq_{cx} Y$

- If $X \sim \exp(\lambda_1)$ and $\mathrm{Y} \sim \exp(\lambda_2)$ and $\lambda_1 \geq \lambda_2$then $X \leq_{cx} Y$

For many distributions, including *uniform, gamma, Lognormal*, and *Beta*, if two random variables have the same mean but one has smaller variance then a **convex ordering exists**.

Evaluating Fixed Schedules

Example: Consider two instances of $1 \mid \mid \sum C_j$ with 2 jobs and random processing times:

Instance 1 Processing Times: $Z_1^{(1)}, Z_2^{(1)}$
Instance 2 Processing Times: $Z_1^{(2)}, Z_2^{(2)}$

where $Z_1^{(1)} \geq_{cx} Z_1^{(2)}$ and $Z_2^{(1)} \geq_{cx} Z_2^{(2)}$.

Total completion time is $\sum C_j^{(1)} = 2Z_1^{(1)} + Z_2^{(1)}$ and $\sum C_j^{(2)} = 2\,Z_1^{(2)} + Z_2^{(2)}$. Therefore:

$$E[\sum C_j^{(1)}] \geq E[\sum C_j^{(2)}]$$

Evaluating Fixed Schedules

Example: Consider two instances of $P2 \mid \mid C_{max}$ with 2 jobs with random processing times:

Instance 1 Processing Times: $Z_1^{(1)}, Z_2^{(1)}$

Instance 2 Processing Times: $Z_1^{(2)}, Z_2^{(2)}$

where $Z_1^{(1)} \geq_{cx} Z_1^{(2)}$ and $Z_2^{(1)} \geq_{cx} Z_2^{(2)}$. The makespan is $C_{max}^{(1)} = \max\{Z_1^{(1)}, Z_2^{(1)}\}$ and $C_{max}^{(2)} = \max\left\{Z_1^{(2)}, Z_2^{(2)}\right\}$.

Since $\max\{a, b\}$ is a convex function it follows:

$$E\left[C_{max}^{(1)}\right] \geq E\left[C_{max}^{(2)}\right]$$