IO543: Scheduling

Lecture 18

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Today

Reminder: Project summaries are due today.

Today we will cover:

- More on single machine scheduling problems
- Expected value of perfect information

Mean Value Problem

Definition: The **mean value problem** is the problem obtained by replacing all random variable with their mean.

<u>Sometimes</u> the optimal solution to the mean value problem is the optimal solution to a stochastic scheduling problem.

Often this is the case if the criteria for the scheduling problem is a linear function of the random variables.

Stochastic $1 \mid \mid \sum w_j C_j$

The stochastic version of $1 \mid \mid \sum C_j$ is solved by the **deterministic counterpart** called the **mean value problem**

$1 \mid \mid \sum w_j C_j$ can also be solved by solving the corresponding mean value problem

Theorem: The weighted shortest <u>expected</u> processing time first (WSEPT) rule is optimal for the stochastic version of $1 \mid \mid \sum w_j C_j$.

Proof left as an exercise

Stochastic $1||L_{max}$

$1||L_{max}$ does do not have a linear objective function:

$$L_{max} = \max_{i \in \{1,\ldots,n\}} L_i \quad \text{where } L_i = C_i - d_i$$

In spite of the nonlinearity the stochastic counterpart of $1||L_{max}$ with random processing times is easy:

Theorem: The earliest due date (EDD) rule is optimal for the stochastic counterpart of $1||L_{max}$.

Proof: Completed in class.

Value of Information

Often it is possible to obtain advance warning about factors that affect manufacturing and service systems:

- Collecting data earlier in a process
- Developing predictive statistical models based on historical data to anticipate random outcomes

Since there is a cost to implementing these measures it is important to be able to measure the *value of information*

Classes of Policies

The time at which random variables are observed is an important characteristic of stochastic scheduling problems:

- In a **clairvoyant** context the decision maker knows the **outcomes before** deciding on the schedule
- In a **non-clairvoyant** context the decision maker **does not know the outcome** in advance.

The expected value of the difference in these two cases is called the **expected value of perfect information (EVPI)**

EVPI

Definition: The difference between the optimal solution to the clairvoyant and the non-clairvoyant problem is the **Expected Value of Perfect Information (EVPI)**

Let $\mathrm{h}(s, \omega)$ denote the criteria for a scheduling problem, given scenario ω and schedule s.

Let $s(\omega)$ denote the optimal (clairvoyant) schedule given scenario ω and s^* the optimal (non-clairvoyant) schedule that minimizes $E_\omega[h(s, \omega)]$.

$$\boldsymbol{EVPI} = E_\omega[h(s^*, \omega)] - E_\omega[h(s(\omega), \omega)]$$

In-Class Exercise

What is the EVPI for the following stochastic instance of $1 \mid \mid \sum w_j C_j$?

Job Weights: $w_1 = 1, w_2 = 3, w_3 = 5$

Processing Time Distributions:

Job 1:

Outcome (ω_1)	1	2
$p(\omega_1)$	1/2	1/2
$t_1(\omega_1)$	1	2

Job 2:

Outcome (ω_2)	1	2
$p(\omega_2)$	2/3	1/3
$t_2(\omega_2)$	2	4

Job 3

Outcome (ω_3)	1
$p(\omega_3)$	1
$t_3(\omega_3)$	3