# IO543: Scheduling

## Lecture 13

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# Today

## Job Shops

# Job Shops

There are many different types of job shops

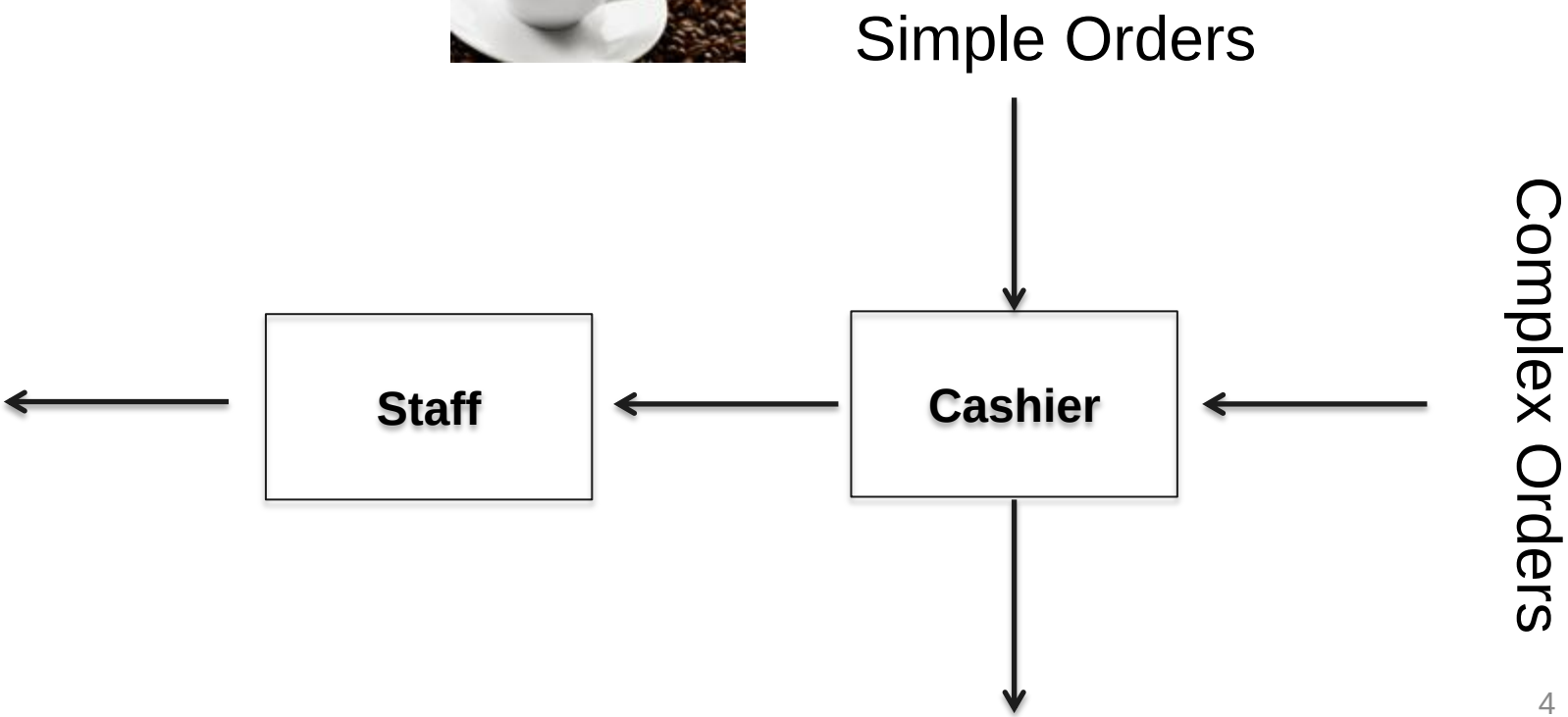
We will focus on:

- $Jm \mid\mid C_{max}$
- $J\,m \mid\mid \sum_j w_j T_j$

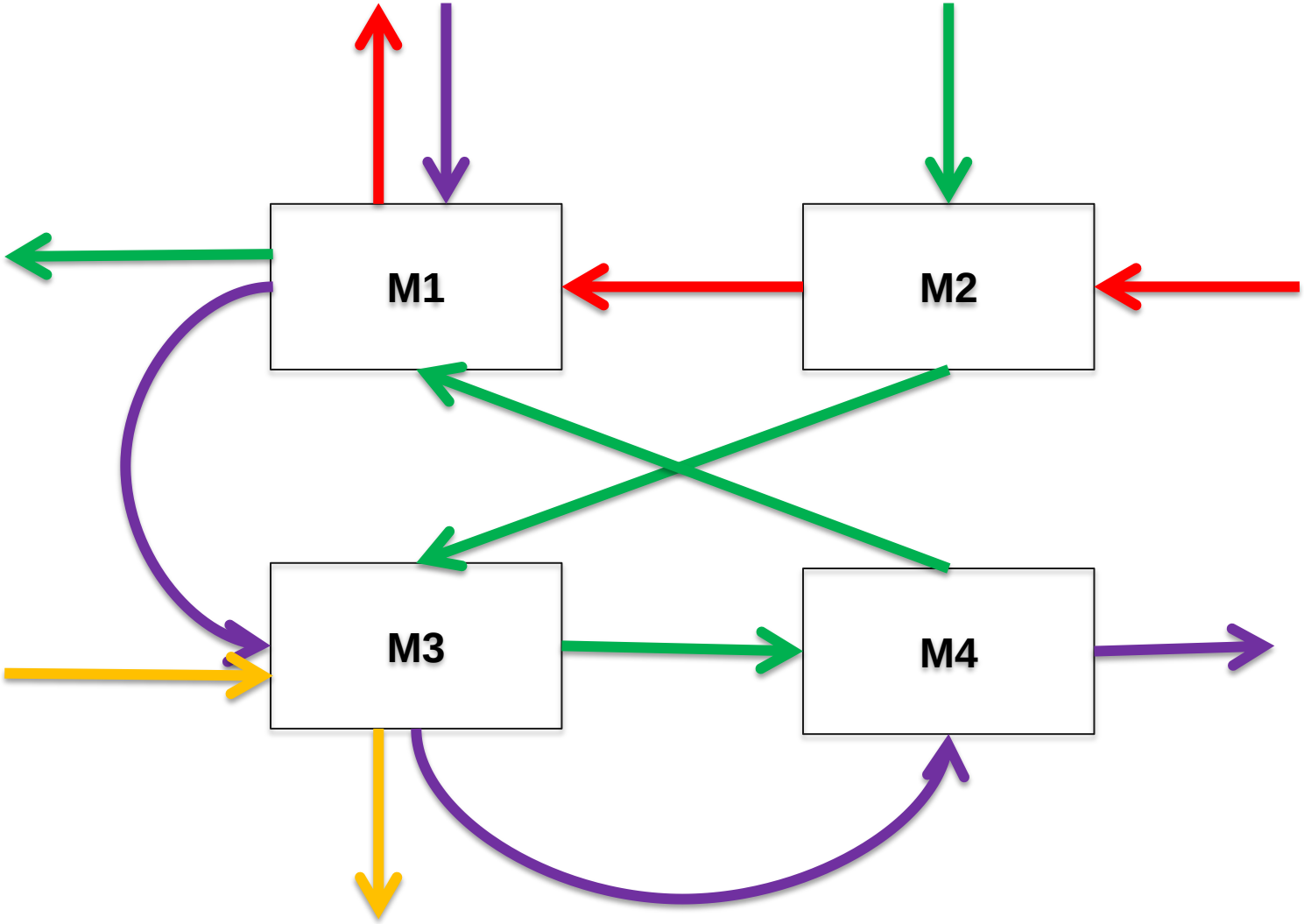
# Job Shop Basics

Job shops are similar to flow shops except jobs vary in machines they visit and the order they visit them

- Coffee Shop:



# Job Shop: Example

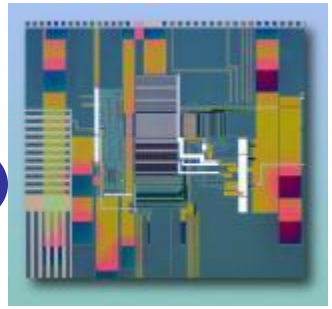


# Example

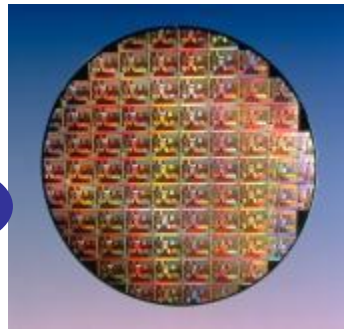


## Semiconductor manufacturing:

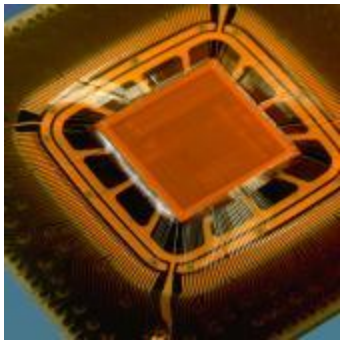
Design



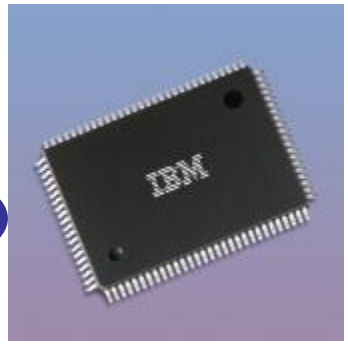
Wafer Fab



Wafer Test



Packaging




Final Test

# “Main Street”



- “Jobs” are sealed pods containing 24 wafers
- Jobs move on an automated material handling system

# Automated Transport

- Jobs are stored in an inventory buffer
- Machines have limited buffer space for waiting jobs

# Furnace Bay

- Furnaces process **batches** of jobs simultaneously

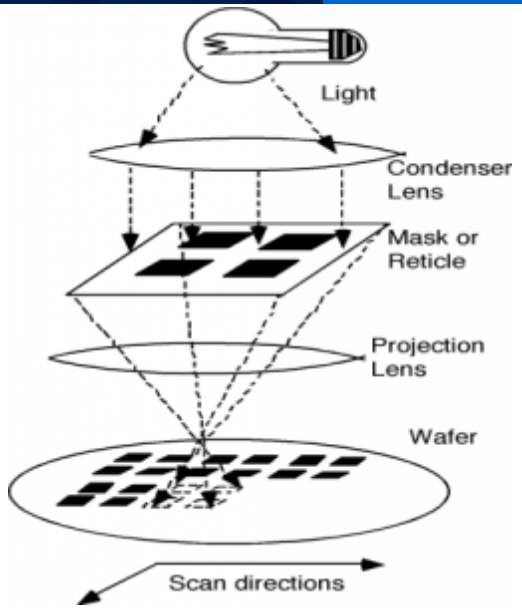
- Question: what is the optimal batch size?

# Photolithography Bay

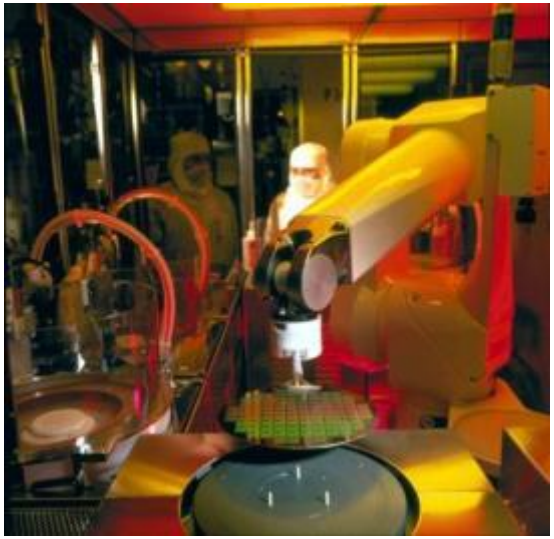
- Photolithography tools are often the **bottleneck** in the fab
- Often there is significant work-in-process (WIP) waiting for photo tools

# Photo Cluster Tool

- Circuits are projected onto wafers via a photolithography process



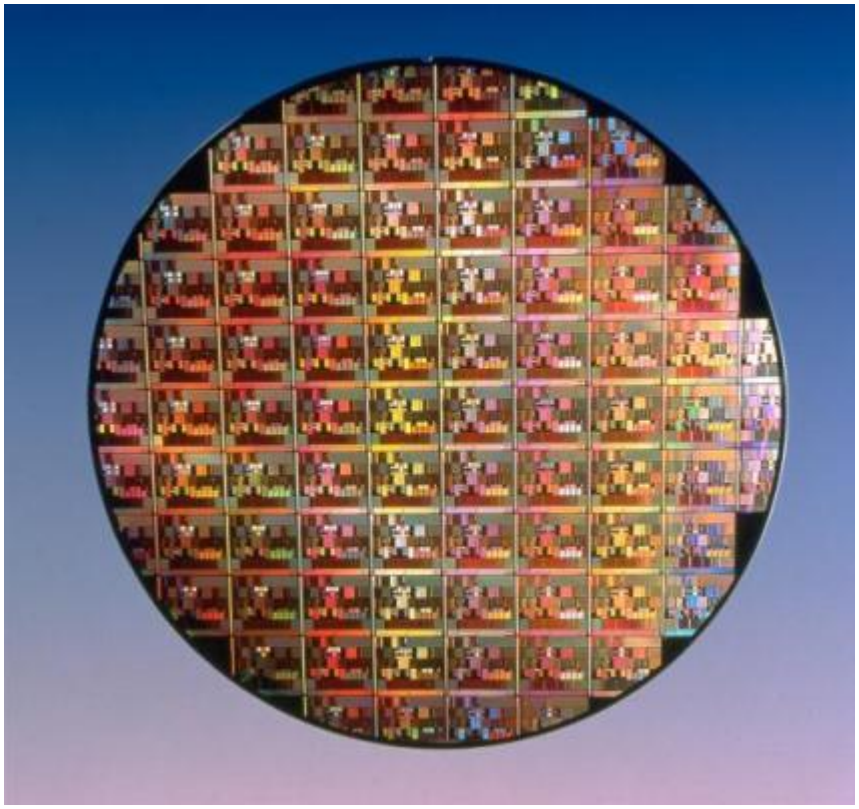

- The photo operation is composed of many sub-operations



# Finished Wafer

- Finished wafers are tested for defects

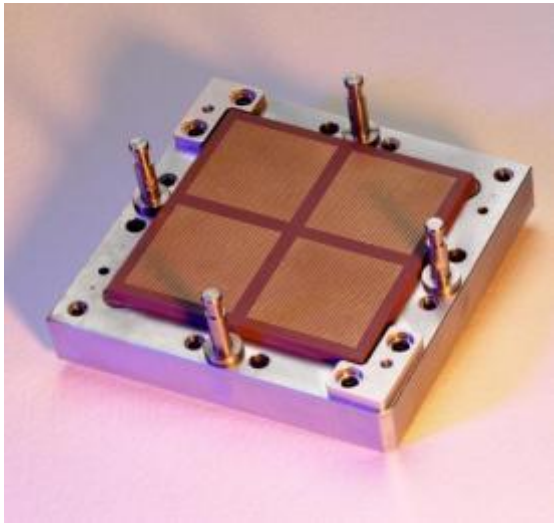
- Wafers are “diced” into chips and sorted based on quality

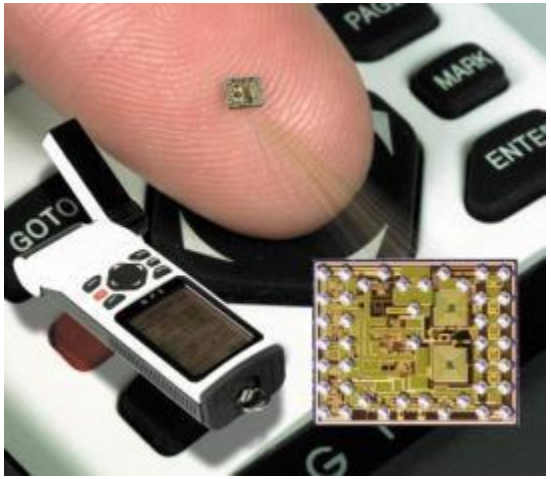


# Finished Products



- Finished products include chips for applications in: servers, wireless devices, game systems…

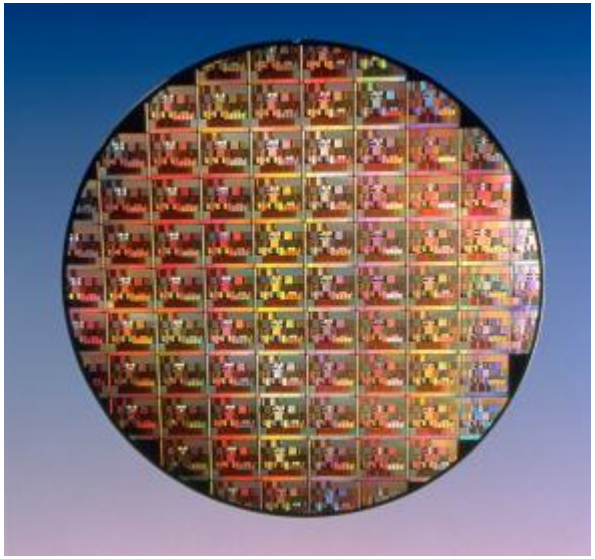




# Automated Semiconductor Fab

IBM Automated Fab (From Sand to Chips):

http://www.youtube.com/watch?v=inoOAOOMjHo&playnext=1&list=PLFC1F76CF83D4033F&feature=results_video



# Job Shop Inputs

A job shop is defined by:

- Number of machines
- Number of jobs
- Sequence of operations for each job (**routing**)
- Processing time for each operation for each job
- Due dates (if tardiness is the criteria)

# Relationship to Flowshops

$J2 \mid \mid C_{max}$ decomposes to $F2 \mid \mid C_{max}$

- Let $J_{1,2}$ be the set of jobs processed on M1 first and $J_{2,1}$ the set processed on M2 first
- $J_{1,2}$ jobs have higher priority on M1; $J_{2,1}$ jobs have higher priority on M2

<u>Solution:</u>

- Treat $J_{1,2}$ as $F2 \mid \mid C_{max}$ with 1st priority on M1
- Treat $J_{2,1}$ as $F2 \mid \mid C_{max}$ with 1st priority on M2

# Example: $J2 \mid \mid C_{max}$



5 jobs, 2 machines:

| Jobs | Machine Routing | Processing Times |
|---|---|---|
| 1 | 1,2 | $p_{11} = 5, p_{21} = 2$ |
| 2 | 1,2 | $p_{12} = 3, p_{22} = 4$ |
| 3 | 2,1 | $p_{13} = 3, p_{23} = 2$ |
| 4 | 2,1 | $p_{14} = 4, p_{24} = 6$ |
| 5 | 2,1 | $p_{15} = 1, p_{25} = 9$ |

Find an optimal schedule.

# $Jm \mid \mid C_{max}$ as a Network



**Network Representation**:

- **Conjunctive** arcs represent **routes** of jobs
  - Arc (i,j) $\longrightarrow$ (k,j) denotes operation (i,j) precedes (k,j)
- **Disjunctive** arcs represent **sequence** of jobs on a machine
  - Arc (i,j) $\dashrightarrow$ (i,k) denotes operation (i,j) precedes (i,k)

# Job Shop as a Directed Network

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## **Disjunctive Network Construction**:

- Add a **start** node *S* and **end** node *E*
- Add a node for every **operation**
- Connect S to first operation of each job and E to the last operation of each job with a **conjunctive arc**
- Add a **conjunctive arc** for each pair of jobs in a routing
- Add a **disjunctive arc** for each pair of jobs on a machine

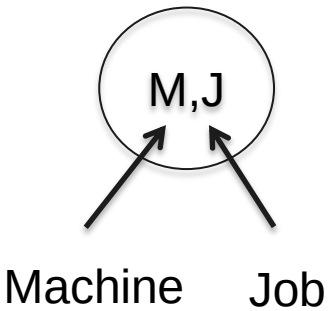
# $J2 \mid \mid C_{max}$ as a Directed Network

Create the network for the following instance of $J2 \mid \mid C_{max}$

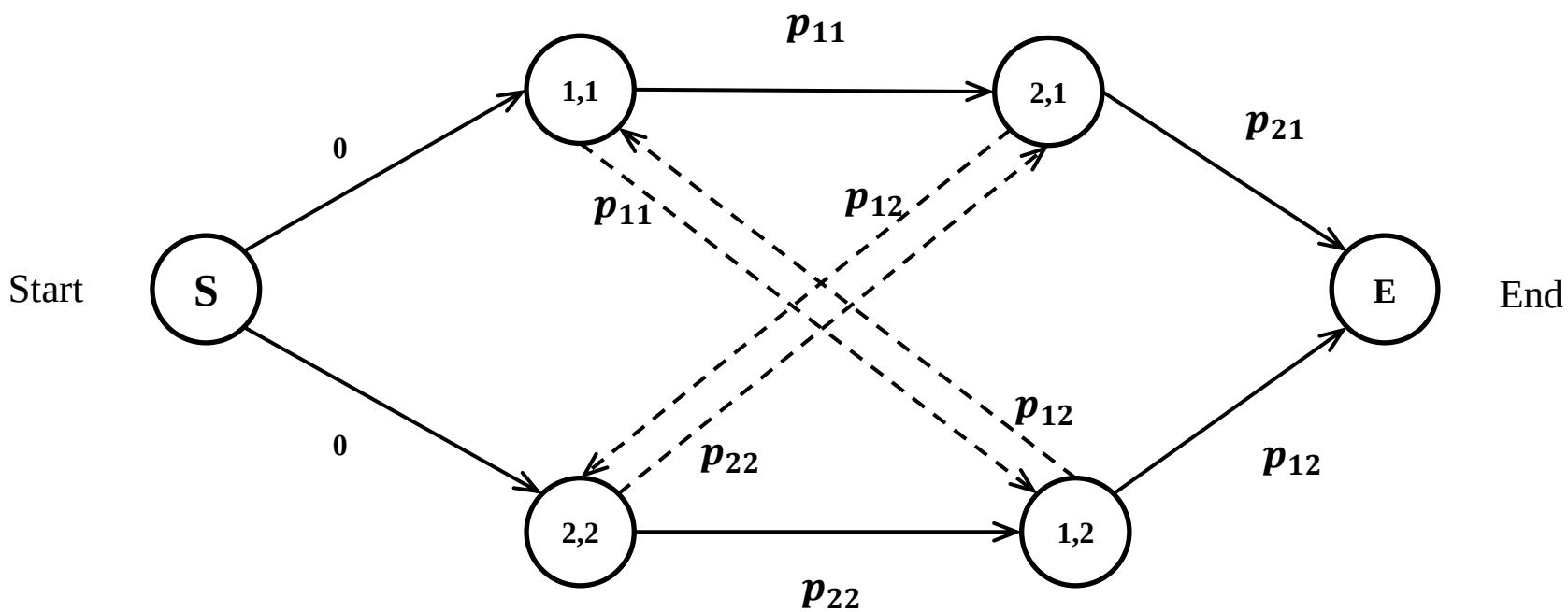
| Jobs | Machine Routing | Processing Times |
|---|---|---|
| 1 | 1,2 | $p_{11} = 2, p_{21} = 1$ |
| 2 | 2,1 | $p_{22} = 3 \;\; p_{12} = 1$ |

What is the optimal schedule?

# Jobs Shop Disjunctive Network




The critical path determines the makespan. The network with the shortest critical path minimizes makespan

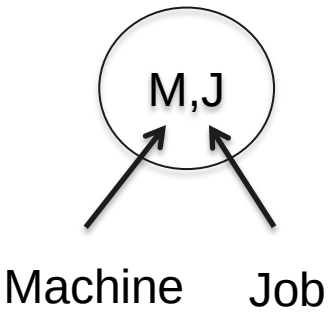


# Jobs Shop Disjunctive Network

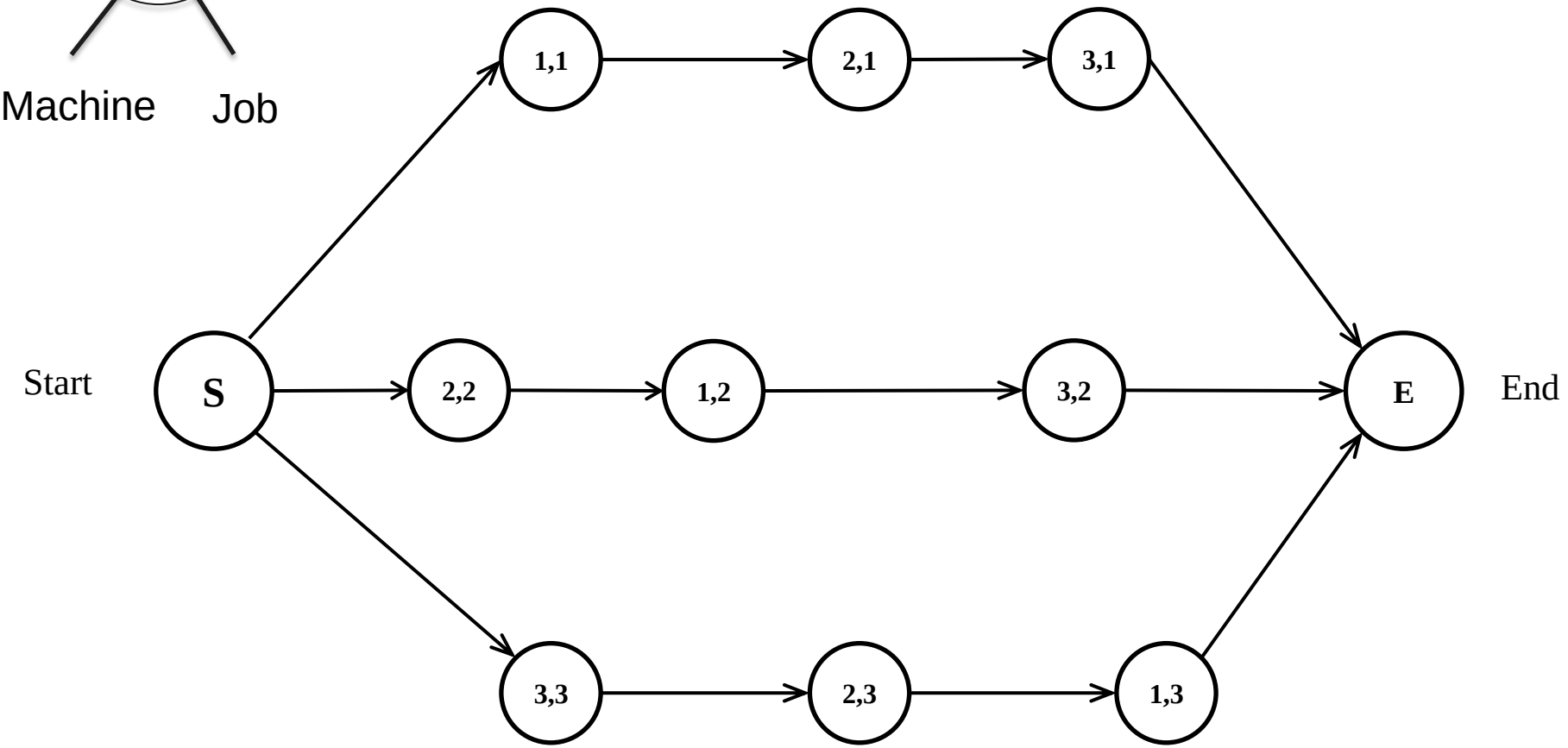
Create the directed graph for the following instance of $J3 \mid \mid C_{max}$

| Jobs | Machine Sequence | Processing Times |
|---|---|---|
| 1 | 1,2,3 | $p_{11} = 4, p_{21} = 3, p_{31} = 2$ |
| 2 | 2,1,3 | $p_{22} = 4, p_{12} = 1, p_{32} = 4$ |
| 3 | 3,2,1 | $p_{33} = 3 \;\; p_{23} = 2, p_{13} = 3$ |

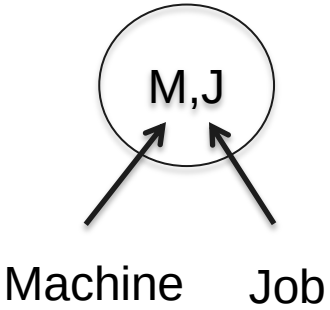
# Jobs Shop Disjunctive Network



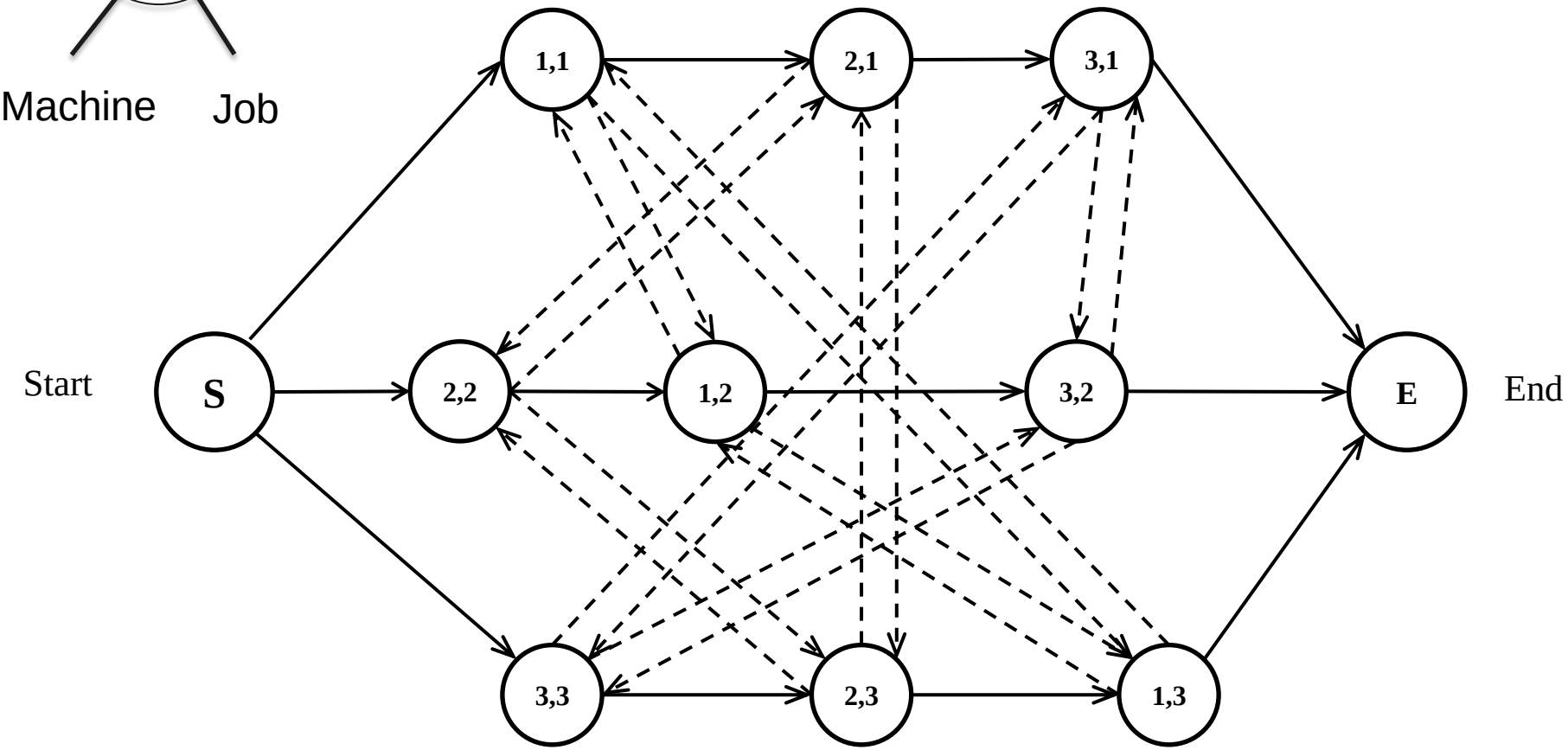

Add the conjunctive arcs first:



# Jobs Shop Disjunctive Network

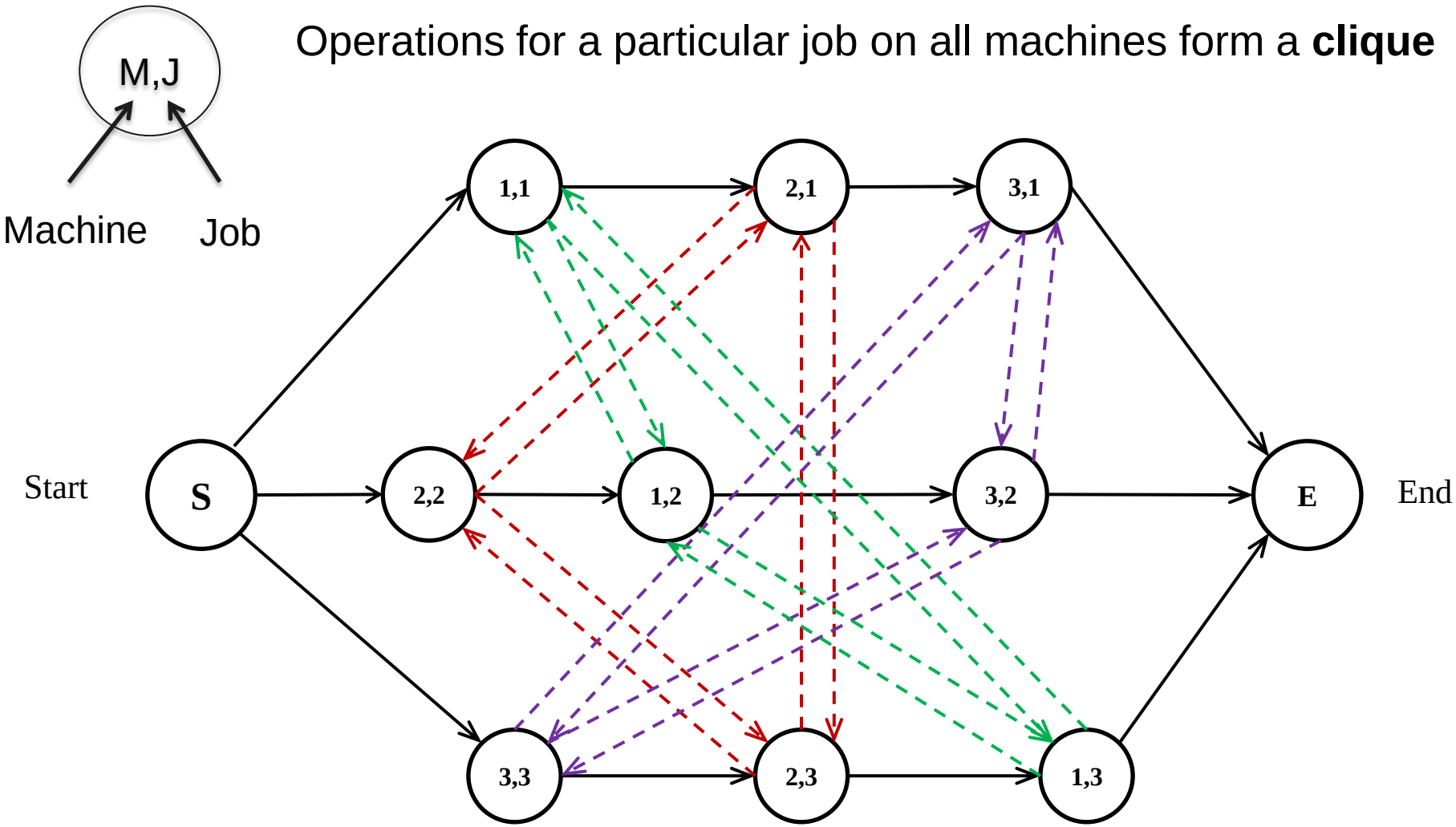



Add the disjunctive for each machine:



# Jobs Shop Disjunctive Network

Operations for a particular job on all machines form a **clique**



# Reading

For the next few classes the relevant reading is Chapter 7 of Pinedo:

- We will cover sections 7.1,7.2, and 7.3
- After Jobs shops we will move on to <u>stochastic scheduling.</u>

# In-Class Assignment



Next time we will have an in-class assignment. To prepare for it watch the following video on the “Job Shop Game”

http://www.youtube.com/watch?v=FSvjEIqSoIw

Come to the next class prepared to play!