IO543: Scheduling

Lecture 14

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Midterm Exam

- The midterm had a total of 50 points
- The mean was 88% and the range was 62-98%
- Average grade: B+
- Solutions posted on Ctools soon

Planning Ahead

After Job Shop Scheduling we will cover topics in Stochastic Scheduling:

- Chapter 9 reviews some important concepts
- We will cover selected topics on stochastic single and parallel machine scheduling

There will be **3 more assignments** tentatively due: March 17, March 31, April 7

Mini Project

In groups of 3-4 pick a scheduling application you are interested in and present to the class

Presentations will be April 7 and 12. The following are guidelines for the presentations:

- Presentations should be 10 minutes in duration
- Everyone in the group should be involved in the presentation
- The presentation should describe a real scheduling problem and/or method and its relationship to topics covered in this class

Due Thursday, March 24: Names of people on your team, a summary (max one page) describing what you plan to cover in your presentation.

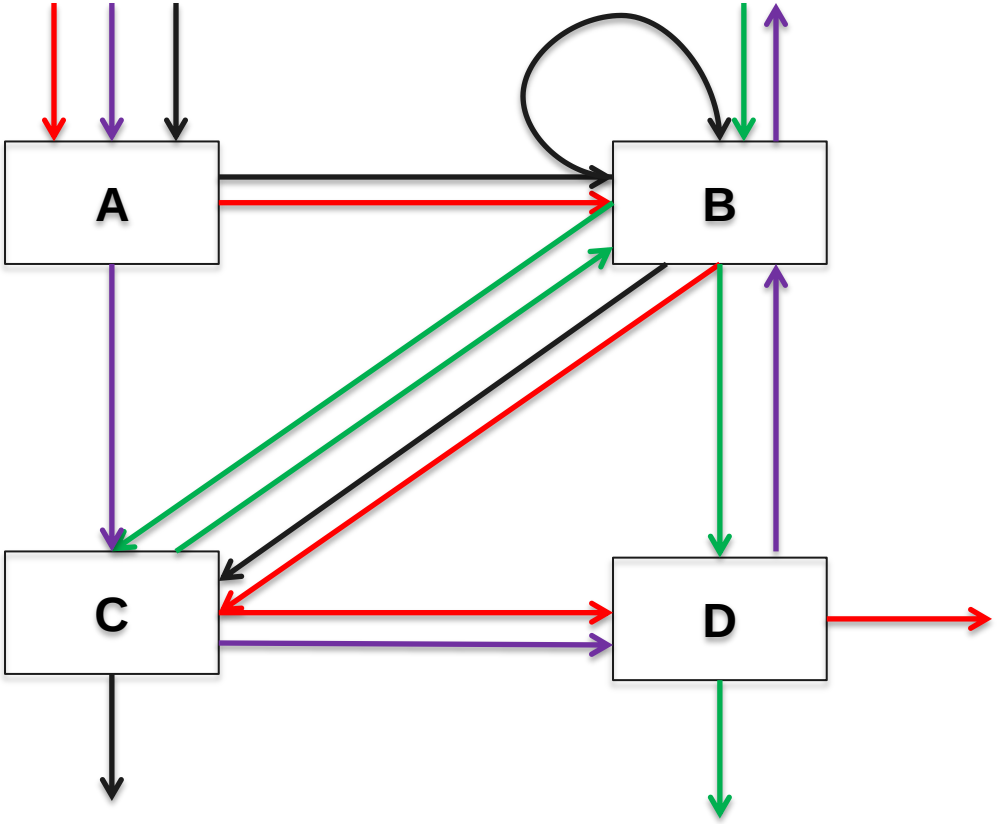
Today

- Job Shop Game**:
  http://public.wsu.edu/~engrmgmt/holt/em530/Docs/JSGInstinfo.htm

**Developed by James Holt at Washington State University

Job Shop Game

Consider a job shop with 4 machines: A, B, C, D and 4 products (jobs): P1, P2, P3, P4



P1 goes: A->B->C->D Out

P2 goes: B->C->B->D Out

P3 goes: A->C->D->B Out

P4 goes: A->B->B->C Out

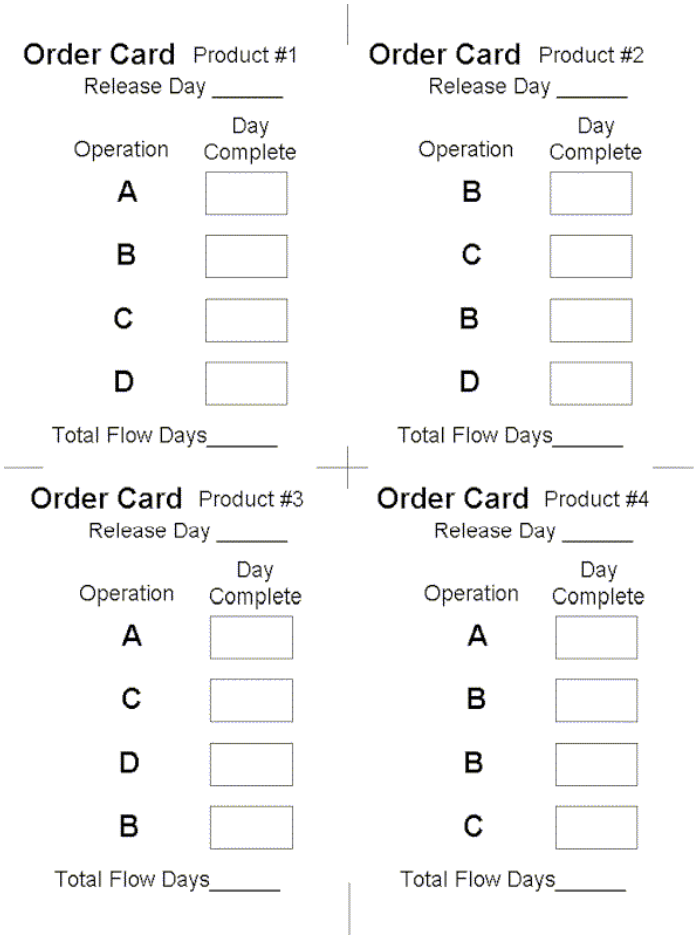
The Playing Cards

Jobs are released into the shop at a rate of **one per day**

Each operation takes **one day**

<u>Playing cards define:</u>

- Routing for each job
- Release day
- Completion time of each operation
- Flow time



Order Card Product #1
Release Day _____

Operation	Day Complete
A	
B	
C	
D	

Total Flow Days_____

Order Card Product #2
Release Day _____

Operation	Day Complete
B	
C	
B	
D	

Total Flow Days_____

Order Card Product #3
Release Day _____

Operation	Day Complete
A	
C	
D	
B	

Total Flow Days_____

Order Card Product #4
Release Day _____

Operation	Day Complete
A	
B	
B	
C	

Total Flow Days_____

Rules of the Game

- Get in groups of 5 and assign one person as **Operator** of each machine (A, B, C, D); Assign one person to be the **Gate Keeper** and **Recorder**

- **At the beginning of each day**, the **Gate Keeper** calls out the day and writes the release day on the newly released job from the top of the shuffled stack of cards

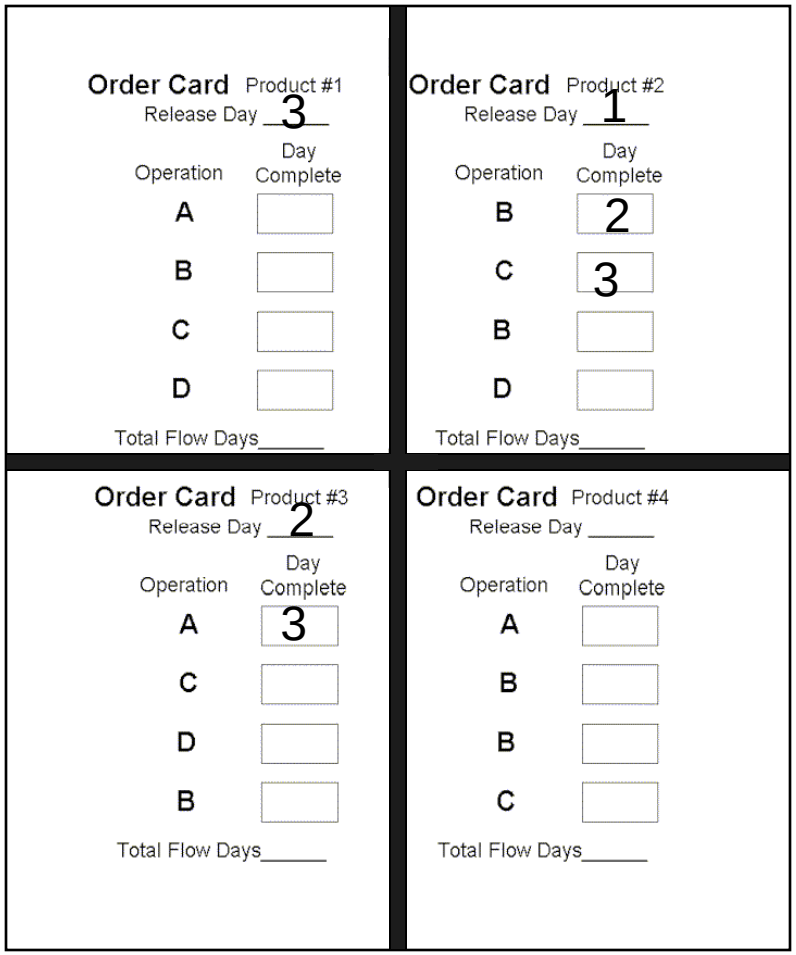
- **At the beginning of each day** each **Operator** writes the day in the appropriate routing box for one order in their possession (**if there is more than one order the Operator selects one**)

- At the end of each day the **Gatekeeper** passes the newly released job to the first **Operator** in the routing, and each **Operator** passes any completed cards to the next **Operator** in the routing

- Each time a job is completed the **Recorder** records the flow time

Note: An Operator can only write a number if they have an order card that needs their operation on that day. They must have the Order Card in their possession at the beginning of the day.

Example



Assume the first card in the shuffled deck is Product 2. The Gate Keeper calls out “Day 1” and writes a “1” on the Product 2 card. No one else has work. The gatekeeper passes her card to Operator B.

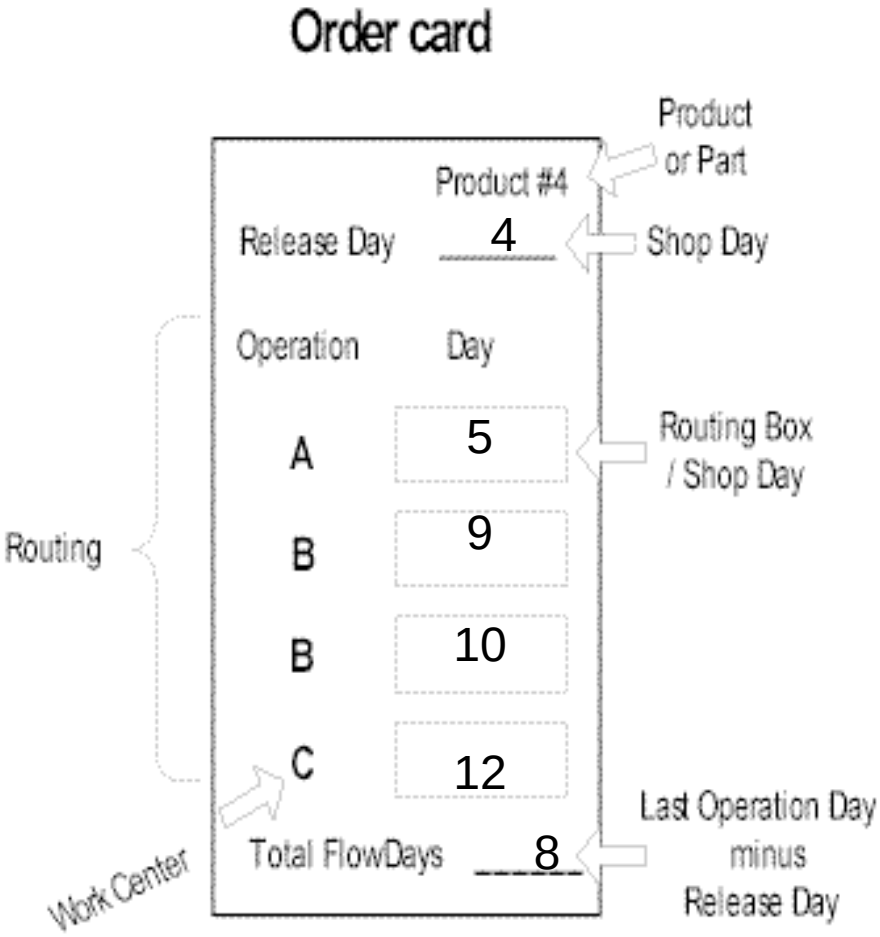
Assume the second card in the deck is Product 3. The Gate Keeper calls out “Day 2” and writes a “2” on the Product 3 Card.

On Day 2, Operator B can work and writes a “2” on the Product 2 card.The gatekeeper passes her Product 3 card to Operator A and Operator B passes her card to Operator C.

Assume the third card in the deck is Product 1. On Day 3, the Gate Keeper calls out “Day 3” and writes “3” on a Product 1 Order Card. On Day 3, both Operators C and A have work….

The Gate Keeper continues to release one card per day until the Order Cards are gone. Play continues until all products are finished.

Calculating Flow Days



Eventually a job will finish and come to the recorder.

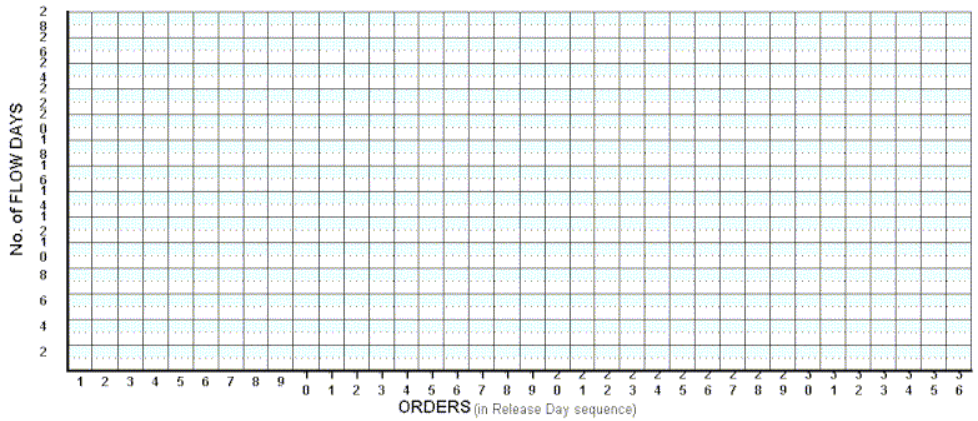
The recorder subtracts the Release Day from the Last Operation to calculate the total flow days for the Product.

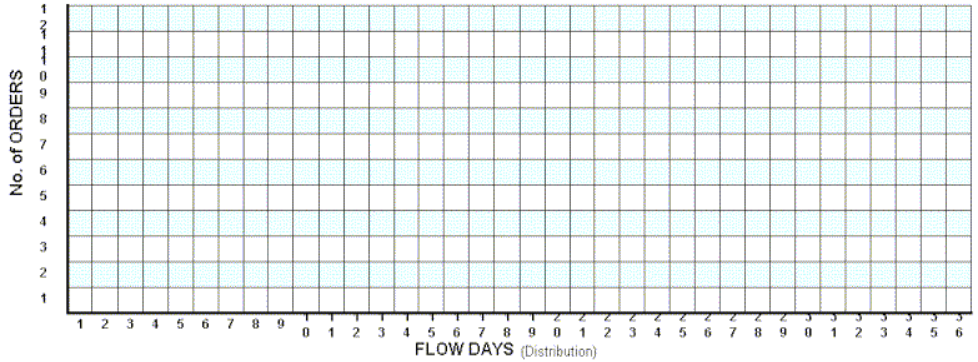
Here, Product #4 was released on Day 4, finished A on day 5, finished first B on day 9, finished second B on day 10, finished C on day 12.

The total flow days = 12 – 4 = 8 days

The flow days number is recorded on two charts

Record Keeper





The top chart is a <u>scatter plot</u> of the **Day Released** versus **Flow days**. Put an "X" for the flow days for each completed order.

The second chart is a <u>histogram</u> of the number of times a particular flow day occurred. Put an "X" in the flow day column each time an order completes with that number of flow days

In-Class Assignment

1) In groups of 5 play the Job Shop Game using randomly shuffled cards

2) Use the results of the first game to come up with idea(s) to **improve the sequence of the jobs**

3) Describe what motivated your idea(s) and the difference you observed in the results for the 2 games. **Draw any relevant conclusions from the differences**.

Hand in the scatter plots and histograms from the two rounds of play and the description from part (3).

Job Shop Disjunctive Program

$$min \quad C_{max}$$

A denotes the set of Arcs and N the set of nodes

subject to

$$y_{kj} - y_{ij} \geq p_{ij}$$ for all $(i,j) \rightarrow (k,j) \in A$

$$C_{max} - y_{ij} \geq p_{ij}$$ for all $(i,j) \in N$

$$y_{ij} - y_{il} \geq p_{il} \text{ or } y_{il} - y_{ij} \geq p_{ij}$$ for all (i,l) and (i,j)

$$y_{ij} \geq 0$$ for all $(i,j) \in N$

Disjunctive Constraints

A set of **disjunctive constraints** requires that a point must satisfy k of m constraints

$$\mathrm{A}^i y \leq b^i, \quad i = 1, \dots, m$$

If k of m must be satisfied:

$$\mathrm{A}^i y \leq b^i + M(1 - x_i) \quad i = 1, \dots, m$$

$$\sum_{i=1}^{m} x_i = k$$

Job Shop Integer Program

$$min \quad C_{max}$$

subject to

$$y_{kj} - y_{ij} \geq p_{ij} \qquad \text{for all } (i,j) \rightarrow (k,j) \in A$$

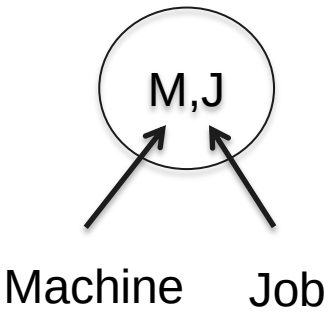
$$C_{max} - y_{ij} \geq p_{ij} \qquad \text{for all } (i,j) \in N$$

$$y_{ij} - y_{il} + M(1-x_{ilj}) \geq p_{il} \qquad \text{for all } (i,l) \text{ and } (i,j),\ i=1,\ldots,m$$

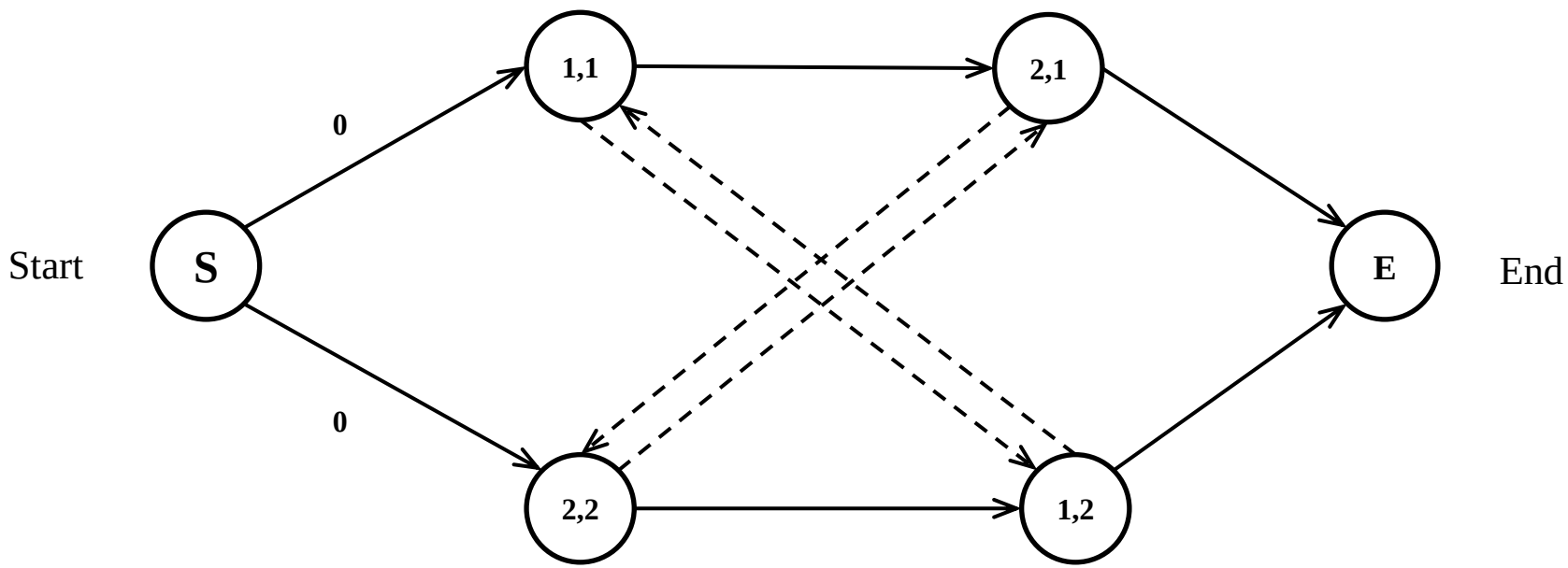
$$y_{il} - y_{ij} + Mx_{ilj} \geq p_{ij} \qquad \text{for all } (i,l) \text{ and } (i,j),\ i=1,\ldots,m$$

$$y_{ij} \geq 0 \text{ for all } (i,j) \in N,\ x_{ijl} \text{ binary for all } (i,j,l)$$

Jobs Shop as a Directed Graph



The network with the shortest critical path minimizes makespan



What is the IP formulation? What is the optimal schedule?

Example of IP Formulation

IP Formulation:

$$\min C_{max}$$

Decision Variables:

$C_{max} =$ makespan

y_{ij}= start time of job j on machine i

x_{ijk}= 1 if job j precedes job k on machine i; 0 otherwise

$$\text{s.t.} \quad y_{21} - y_{12} \geq p_{11}$$
$$y_{12} - y_{22} \geq p_{22}$$
$$C_{max} - y_{21} \geq p_{21}$$
$$C_{max} - y_{12} \geq p_{12}$$
$$y_{12} - y_{11} + M(1 - x_{112}) \geq p_{11}$$
$$y_{11} - y_{12} + Mx_{112} \geq p_{12}$$
$$y_{22} - y_{21} + M(1 - x_{212}) \geq p_{21}$$
$$y_{21} - y_{22} + Mx_{212} \geq p_{22}$$
$$y_{ij} \geq 0 \; for \; all \; i, j, \;\; x_{ijk} \; binary \; for \; all \; i, j, k$$