Question from Lecture 6:

Use the DP Algorithm to solve the following instance of $1||\Sigma\, T_j$

jobs	1	2	3	4	5
p_j	121	79	147	83	130
d_j	260	266	266	336	337

Solution:

The jobs are already sorted in increasing order of due date so we are reading to proceed with the dynamic programming algorithm.

Iteration 1:
Job 3 has the highest processing time, so k=3 and $\delta = 0, \dots, n-k = 0,1,2$

$$V(\{1,2,\dots,5\}, 0) = min_{\delta \in \{0,1,2\}} \begin{cases} V(J(1,3,3),0) + (C_3(0) - d_3)^+ + V(J(4,5,3), C_3(0)) \\ V(J(1,4,3),0) + (C_3(1) - d_3)^+ + V(J(5,5,3), C_3(1)) \\ V(J(1,5,3),0) + (C_3(2) - d_3)^+ + V(\phi, C_3(2)) \end{cases}$$

We start by evaluating the tardiness for job 3 for the 3 different possible choice of δ:

$C_3(0) = \sum_{j=1}^{3} P_j = 347$, $C_3(1) = \sum_{j=1}^{4} P_j = 430$, $C_3(2) = \sum_{j=1}^{5} P_j = 560$

Now, we will evaluate the optimal value functions requires for each of the three possible choices of δ:

$V(J(1,3,3),0) = (121-260)^+ + (200-266)^+ = 0$ (Not that J(1,3,3) is the set of jobs {1,2} and either sequence {1,2} or {2,1} has 0 total tardiness)

$V(J(4,5,3),347) = (430-336)^+ + (560-337)^+ = 94 + 223 = 317$ (Again there are two possible sequences and the minimum tardiness is obtained with sequence {4,5})

$V(J(1,4,3),0) = (121-260)^+ + (200-266)^+ + (283-366)^+ = 0$ (Here there are 6 possible sequences. Evaluating them leads to two optimal sequences: {1,2,4} or {2,1,4})

$V(J(1,5,3),0) = 76$ (Obtained by sequence {1,2,4,5} or {2,1,4,5})
Note that larger sets like this can be broke down again, J_5 has the highest process time in {1,2,4,5}, $\delta = 0$, therefore $V(J(1,5,3),0) = V(J(1,2,4),0) + (C_5(2) - d_5)^+ + V\left(\phi, C_5(0)\right) = 76$.

Now, putting everything to getter we can compute the optimal value function for the complete set of jobs:

$$V(\{1,2,\ldots,5\},0) = min\begin{Bmatrix} 0+81+317 \\ 0+164+223 \\ 76+294+0 \ * \end{Bmatrix} = 370$$

Where the minimum tardiness of 370 is obtained by sequences {1,2,4,5} and {2,1,4,5}.