Example from Lecture 18:

Let θ be the set of schedules:

$$\theta = \begin{cases} 1 \to 2 \to 3 \\ 1 \to 3 \to 2 \\ 2 \to 1 \to 3 \\ 2 \to 3 \to 1 \\ 3 \to 1 \to 2 \\ 3 \to 2 \to 1 \end{cases} \Rightarrow \begin{cases} 1 \to 2 \to 3 \\ 1 \to 3 \to 2 \\ 2 \to 1 \to 3 \end{cases} \Rightarrow There\ are\ 3\ distinct\ schedules$$

Let set *I(j,s)* be the set of jobs including job *j* and its predecessors according to schedule s$\in$ θ . Now we can write the problem as:

$$s^* = \text{argmin}_{S\in\,\theta}\{\textstyle\sum_{j=1}^{3} E[C_j(s)]\}$$

where $E\left[C_j(s)\right] = \sum_{i\,\epsilon\,I(j,s)} E_\omega[t_i(\omega)]$

Consider schedule s= $1 \to 2 \to 3$:

$$E[C_1(s)] = E_\omega[t_1(\omega)] = {}^{3}/_{2}$$
$$E[C_2(s)] = E_\omega[t_1(\omega) + t_2(\omega)] = {}^{8}/_{3}$$
$$E[C_3(s)] = E_\omega[t_1(\omega) + t_2(\omega) + t_3(\omega)] = {}^{34}/_{6}$$

And therefore $E\left[C_j(s)\right] = 59/6$

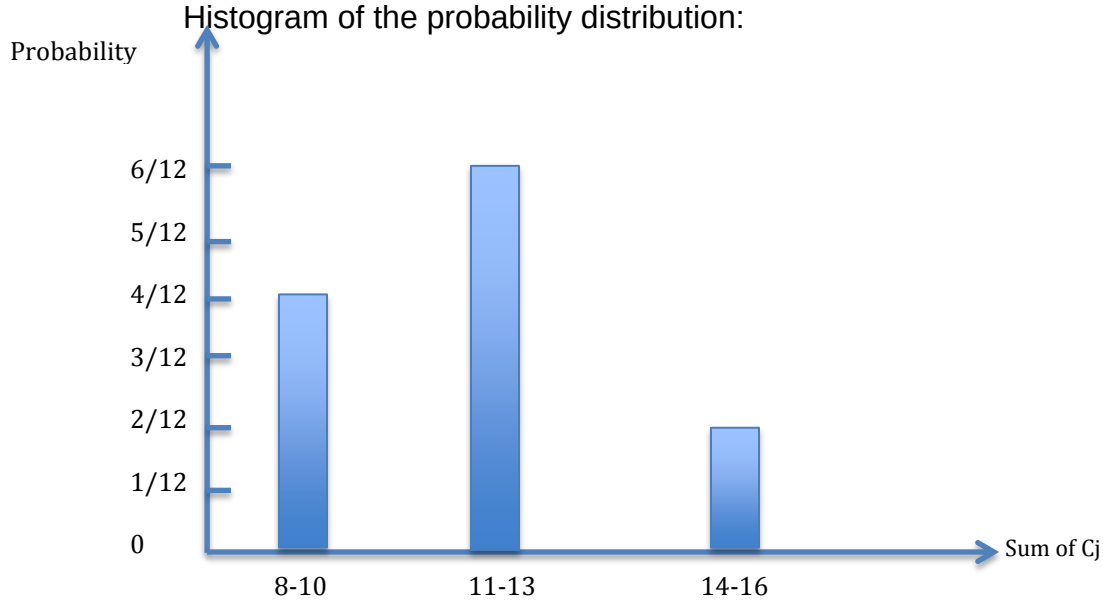
Now doing the same for the remaining two schedules will yield the schedule with lowest cost. Alternatively, we can just recognize the expectation of the sum of random variables is the sum of the expectations. Therefore the stochastic version of $1||\sum C_j$ can be solved by solving the mean value problem:

Job	1	2	3
$E[t_i]$	3/2	8/3	3/2

Thus the Shortest Expected Process Time (ESPT) sequence is optimal and the optimal schedule for this problem is 1→3→ 2 or 3→1→ 2

Example from Lecture 18:

To determine the probability distribution of $\sum C_j$ for schedule $1 \rightarrow 2 \rightarrow 3$, we need to enumerate all the possible values of $\sum C_j$ under each scenario.

Scenario	Outcomes of random variables	Probability	$\sum C_j$
1	(1,2,1)	(1/3)(2/3)(1/2)=1/6	8
2	(1,2,2)	(1/2)(2/3)(1/2)=1/6	9
3	(1,4,1)	(1/2)(1/3)(1/2)=1/12	12
4	(1,4,2)	(1/2)(1/3)(1/2)=1/12	13
5	(2,2,1)	(1/2)(2/3)(1/2)=1/6	11
6	(2,2,2)	(1/2)(2/3)(1/2)=1/6	12
7	(2,4,1)	(1/2)(1/3)(1/2)=1/12	15
8	(2,4,2)	(1/2)(1/3)(1/2)=1/12	16

Histogram of the probability distribution: