Lecture 19 Example: Compute the expected value of perfect information (EVPI) for the following stochastic instance of $1 \mid \mid \sum w_j C_j$.

Job Weights: $w_1 = 1,\ w_2 = 3,\ w_3 = 5$

Processing Time Distributions:

Job 1:

Outcome (ω_1)	1	2
$p(\omega_1)$	1/2	1/2
$t_1(\omega_1)$	1	2

Job 2:

Outcome (ω_2)	1	2
$p(\omega_2)$	2/3	1/3
$t_2(\omega_2)$	2	4

Job 3

Outcome (ω_3)	1
$p(\omega_3)$	1
$t_3(\omega_3)$	3

There are 4 scenarios:

$\omega = 1$: (1,2,3) w.p. 2/6

$\omega = 2$: (1,4,3) w.p. 1/6

$\omega = 3$: (2,2,3) w.p. 2/6

$\omega = 4$: (2,4,3) w.p 1/6.

Let s^*denote the optimal schedule that minimizes expected weighted total completion time for the stochastic version of $1 \mid \mid \sum w_j C_j$. From class we know that we can solve this problem by solving the *deterministic equivalent* problem, called the *mean value problem*, by replacing random variables with their mean.

Mean Value Problem:

Jobs	1	2	3
$E[t_j]$	3/2	8/3	3
w_j	1	3	5
w_j/p_j	2/3	9/8	5/3

It follows that $s^* = J3 \rightarrow J2 \rightarrow J1$

To compute EVPI, first compute the expected value of the optimal solution to the stochastic instance of $1 \mid \mid \sum w_j C_j$:

$$h(s^*, 1) = (5)(3) + (3)(3+2) + (1)(3+2+1) = 36$$

$$h(s^*, 2) = (5)(3) + (3)(3+4) + (1)(3+4+1) = 44$$

$$h(s^*, 3) = (5)(3) + (3)(3+2) + (1)(3+2+2) = 37$$

$$h(s^*, 4) = (5)(3) + (3)(3+4) + (1)(3+4+2) = 45$$

It follows $E_\omega[h(s^*, \omega] = (\frac{2}{6})h(s^*, 1) + \left(\frac{1}{6}\right) h(s^*, 2) + \left(\frac{2}{6}\right) h(s^*, 3) + \left(\frac{1}{6}\right) h(s^*, 4) = 39\frac{1}{6}$

Next, we must solve each scenario problem for $h(s(\omega), \omega)$ for $\omega = 1,2,3,4$.

For $\omega = 1$: $\frac{w_1}{p_1} = 1, \frac{w_2}{p_2} = \frac{3}{2}, \frac{w_3}{p_3} = \frac{5}{3} \implies s(1) = J3 \to J2 \to J1 \; and \; h(s(1), 1) = (5)(3) + (3)(3+2) + (1)(3+2+1) = 36$

For $\omega = 2$: $\frac{w_1}{p_1} = 1, \frac{w_2}{p_2} = \frac{3}{4}. \frac{w_3}{p_3} = \frac{5}{3} \implies s(2) = J3 \to J1 \to J2 \; and \; h(s(2), 2) = (5)(3) + (1)(3+1) + (3)(3+1+4) = 43$

For $\omega = 3$: $\frac{w_1}{p_1} = \frac{1}{2}, \frac{w_2}{p_2} = \frac{3}{2}. \frac{w_3}{p_3} = \frac{5}{3} \implies s(3) = J3 \to J2 \to J1 \; and \; h(s(3), 3) = (5)(3) + (3)(3+2) + (1)(3+2+2) = 37$

For $\omega = 4$: $\frac{w_1}{p_1} = \frac{1}{2}, \frac{w_2}{p_2} = \frac{3}{4}. \frac{w_3}{p_3} = \frac{5}{3} \implies s(4) = J3 \to J2 \to J1 \; and \; h(s(4), 4) = (5)(3) + (3)(3+4) + (1)(3+4+2) = 45$

It follows that $E_\omega[h(s(\omega), \omega] = (\frac{2}{6})h(s(1), 1) + \left(\frac{1}{6}\right) h(s(2), 2) + \left(\frac{2}{6}\right) h(s(3), 3) + \left(\frac{1}{6}\right) h(s(4), 4) = 39.$

Therefore the EVPI = $39\frac{1}{6} - 39 = \frac{1}{6}$ which is less that 0.5% of the total expected weighted completion time for the optimal solution to the stochastic instance of $1 \mid \mid \sum w_j C_j$.