Operations Research: Using Math, Statistics, and Data for the Common Good

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What is Operations Research?

Operations Research (OR) is a field that uses math and statistics for decision-making in many contexts, including healthcare, mobility & transportation, public policy, supply chain management, energy & sustainability, and many others. It integrates mathematics, statistics, and computer science, to address human-centered societal problems.

Putting Theory into Practice

Operations Researchers are needed in a wide range of fields:

- Aerospace
- Business
- Consulting
- Energy
- Finance
- Healthcare
- Manufacturing
- Robotics
- Transportation

Three applications of Operations Research in Healthcare

1. Radiation therapy for cancer treatment
2. Organ transplantation
3. Vaccine purchasing

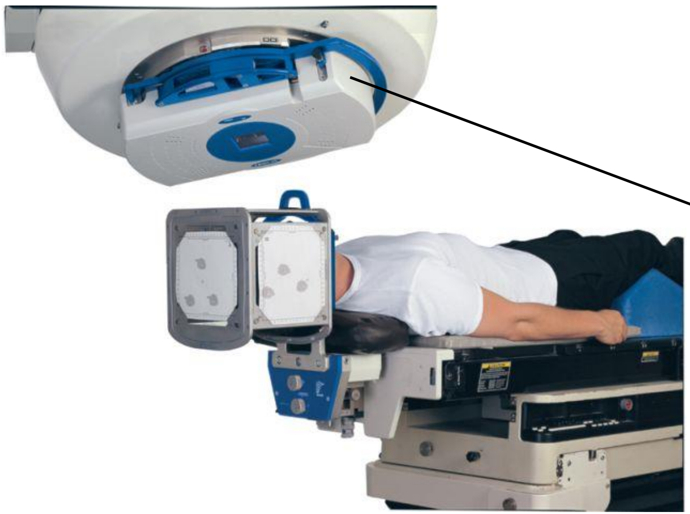
Example 1: Optimizing Radiation Treatment

- External beam radiation is passed through the body harming cancerous and healthy tissue
- Objective: minimize damage to healthy tissue while delivering required dose to cancer tissue

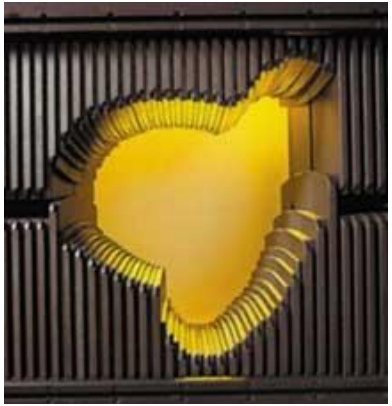
Bahr et al. 1968. “The Method of Linear Programming Applied to Radiation Treatment Planning.” *Radiology*. 91; 686-693.

Radiation is delivered via a rotating gantry with a multi-leaf collimator

Rotating Gantry

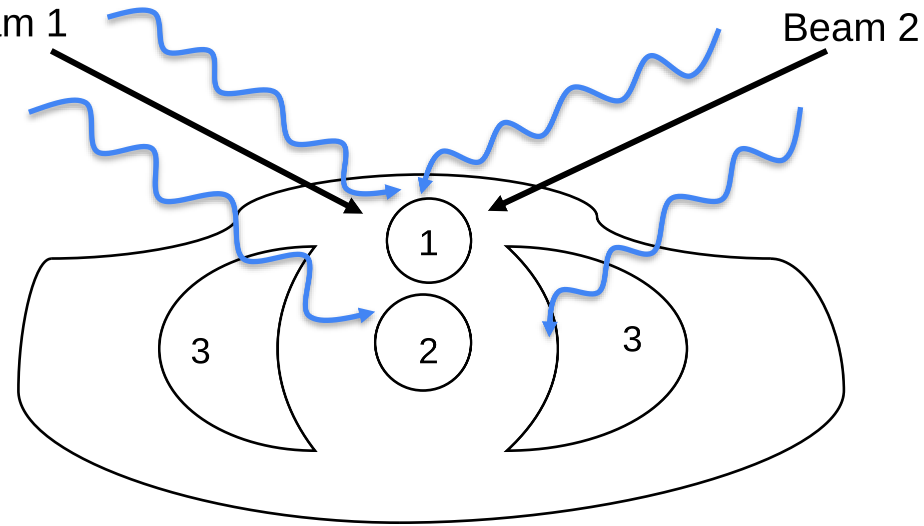


Multileaf Collimator



2-Beam Problem

1. Tumor
2. Spine
3. Brain



Linear Programming Model

Decision Variables: Exposure times for beams 1 and 2 (x_1, x_2)

Area	Dose Absorbed per millisecond: Beam 1 Dose	Dose Absorbed per millisecond: Beam 2 Dose	Restriction on Dosage in Kilorads
Brain	$0.4\, x_1$	$0.5\, x_2$	Minimize
Spine	$0.3\, x_1$	$0.1\, x_2$	$\leq$ 2.7
Tumor	$0.5\, x_1$	$0.5\, x_2$	$=$ 6
Center of tumor	$0.6\, x_1$	$0.4\, x_2$	$\geq$ 6

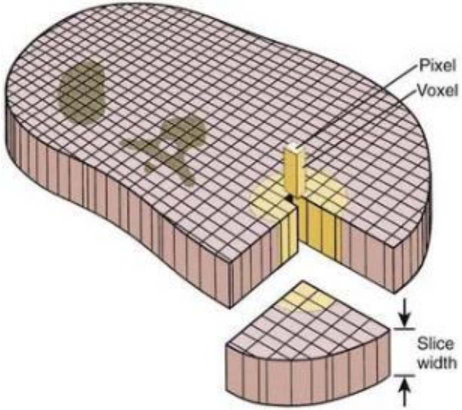
Linear Programming Model

$$Min \ \sum_{\ell \in L} G_\ell(z)$$

$Subject\ to$:

$$z_j = \sum_{k \in K} D_{kj} x_k, \qquad for\ all\ j\ in\ V$$

$$x_k \geq 0, \ \ k \in K, \ \ z_j \geq 0, \ j \in V$$



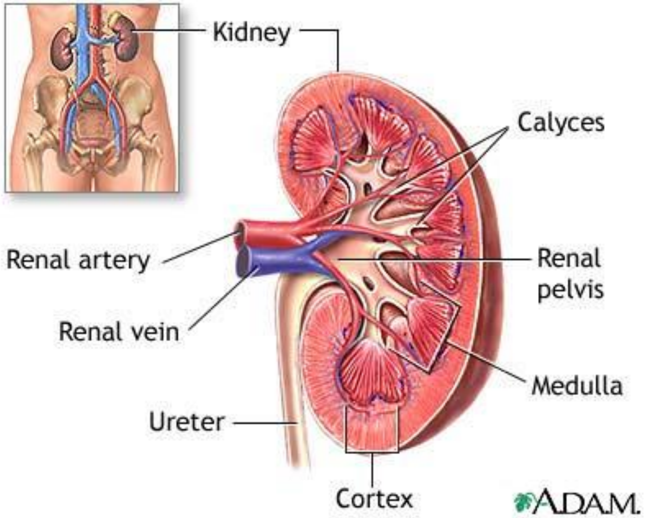
z_j: the dose delivered to voxel $j \in V$

x_k: the duration of beam $k \in K$

Example 2: Kidney Disease

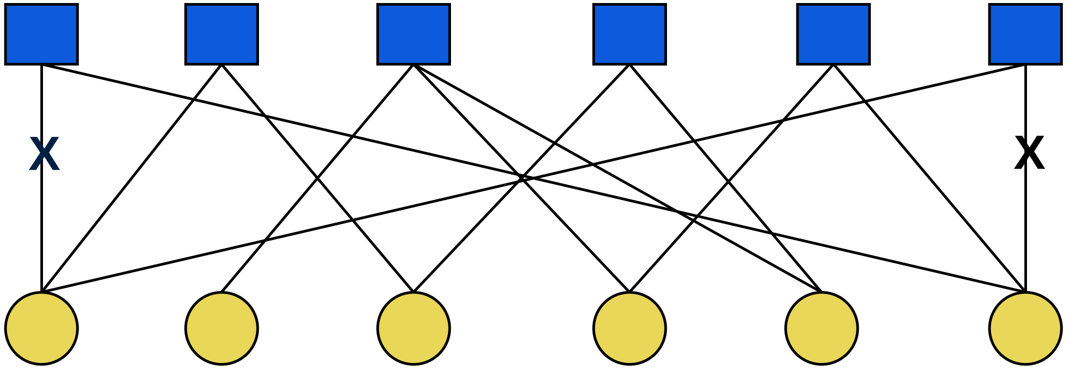
- Principal treatment options:
  - Dialysis (home or clinic)
  - Transplant (live or deceased donor)

- More than 350,000 people are on dialysis and 80,000 waiting for transplant



Kidney Exchange

Donors

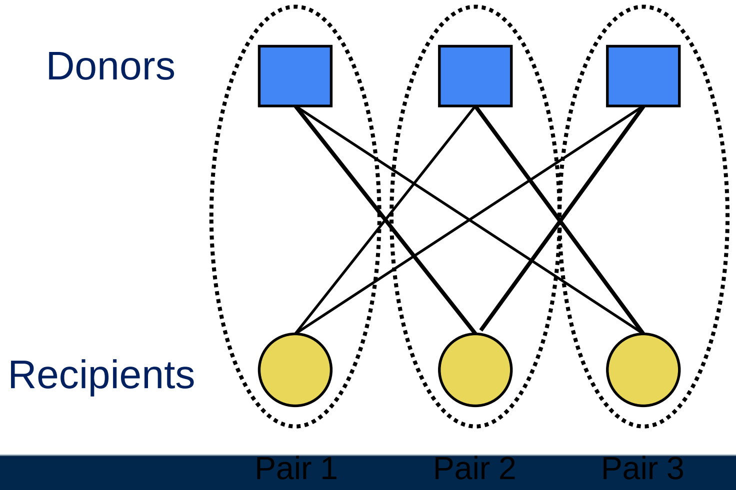


Recipients

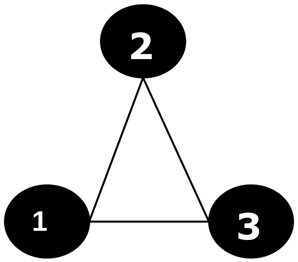
Paired Matching

- A paired exchange allows only two-way exchanges
- Paired matching reduces the chances of a “scuttled” exchange

Assignment



Matching Graph



Paired Kidney Exchanges

Figure 1. Graph Theory Model of Donor/Recipient Nodes, With Links Indicating Compatible Matches

Segev, D, Gentry, S.E., Warren, D.S, Reeb, Montgomery, RA, 2005. “Kidney Paired Donation and Optimizing the Use of Live Donor Organs.” *JAMA.* 293(15), 1883-1890.

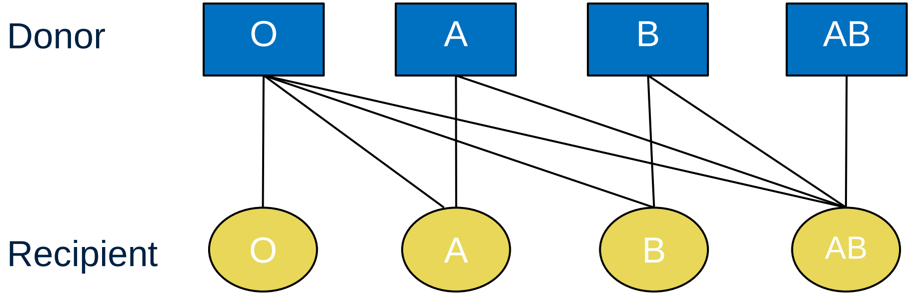
Criteria (Edge Weights)

- Number of matches
- Number of priority matches
- Immunologic concordance
- Travel requirements

Constraints (Edges)

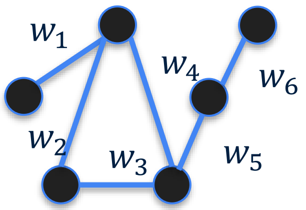
- Compatibility is determined by two primary factors:
  - Blood type
  - Tissue antibodies

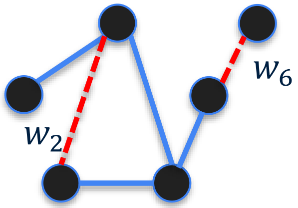
- Blood type compatibility



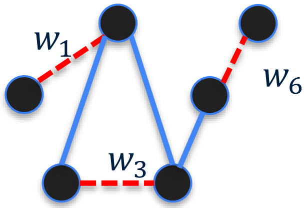
Matching Problems

Given a graph $G(V,E)$ a *matching* is a set of pairwise nonadjacent edges.





2 matches



3 matches

A *maximal edge-weight matching* is a set of non-adjacent edges with maximum total weight among all matches.

Maximum Edge Weight Matching

A matching problem for a graph $G(V, E)$ can be expressed as an *integer program*

$$Max \;\; \sum_{e \in E} w_e x_e$$

$$Subject\ to:$$

$$\sum_{e \sim v} x_e \leq 1, \;\; for\ all\ v \in V$$

$$x_e \in \{0,1\}, \;\; for\ all\ e \in E$$

Edmonds, J. 1965. "Paths, trees, and flowers," *Canadian J. Math*. 17; 449–467.

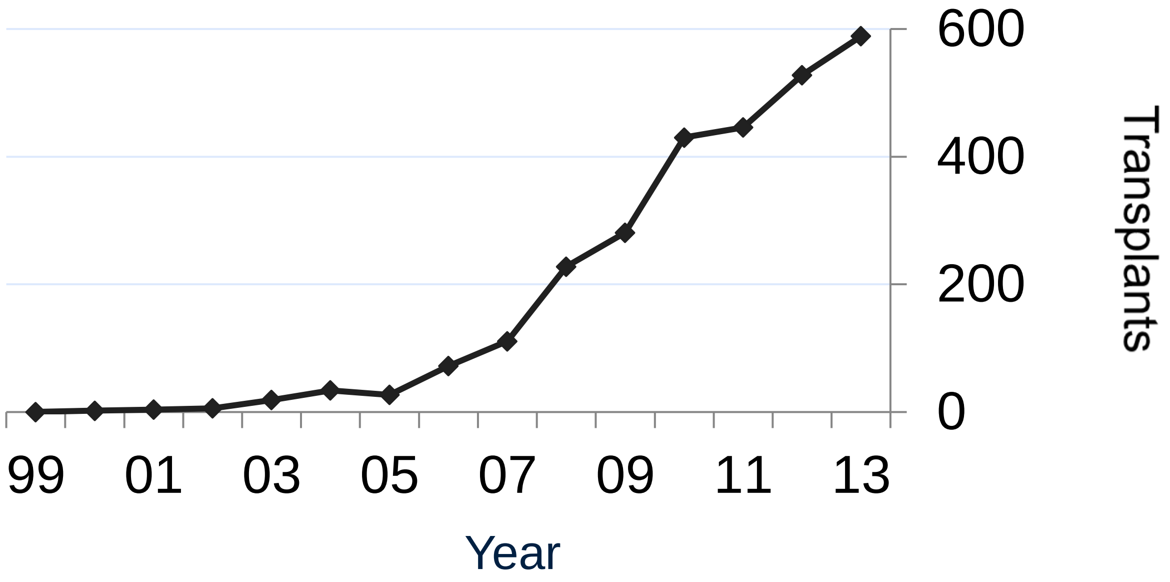
Factors that influence vertex and edge weights

- In a vertex-weighted graph with positive weights, any matching with maximum vertex weight has maximum cardinality

- A maximum edge weight matching could have **half as many** edges as a maximum cardinality matching
  - The ratio can be bounded by controlling : $\max_i w_i - \min_i w_i$

Gentry, S., Michael, T.S., Segev, D. "Maximum Matching in Graphs for Allocating Kidney Paired Donation," Technical Report

Kidney Exchange Impact



From 1 in 1999, to nearly 600 in 2013, KPD now comprises 10% of living kidney donations*

*Figure courtesy of Sommer Gentry, US Naval Academy; www.optimizedmatch.com

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Example 3: Vaccine Purchasing

Problem: How much COVID-19 vaccine should a health system buy for next year?

Tradeoff:

- Ordering too much wastes money
- Ordering too little leaves some patients unvaccinated

Decision Problem Properties:

- Demand for vaccines is unknown in advance
- One-time decision because orders are placed in advance

Newsvendor Problem

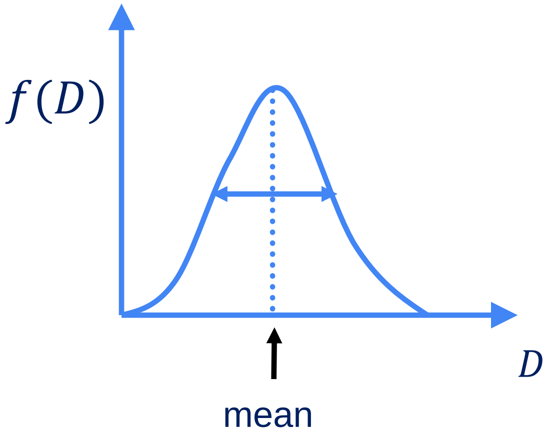
Given the uncertainty in demand for newspapers (vaccine doses), what is the *optimal* purchase?

<u>Model Parameters:</u>

C^U = "underage" cost per item

C^O = "overage" cost per item

D = random variable for demand →



P = purchase quantity

Optimization Model

$$Expected\ Cost\ Z = \min\{E_D[C^U Max(0, D - P) + C^O Max(0, P - D)]\}$$

"Underage"

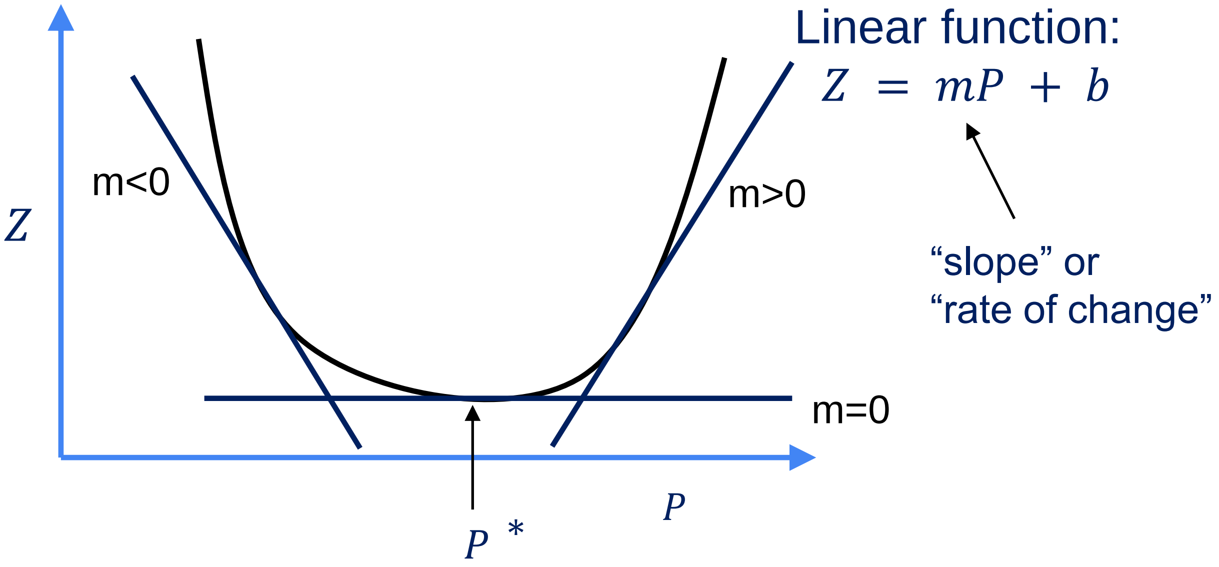
"Overage"

Uncertainty makes things difficult

How can we find P that minimizes expected cost?

What is the Optimal Purchase Quantity?

Find P that satisfies "first-order optimality conditions" for a convex minimization problem



First Order Optimality Condition

- The optimal solution occurs when the 1st derivative of the objective function is zero

$$Z = \min\{ E_D[C^U Max(0, D - P) + C^O Max(0, P - D)]\}$$

$$\boxed{\frac{dZ}{dP} = C^U \Pr(D - P > 0) - C^O \Pr(P - D > 0)}$$

$$First\ Order\ Optimality\ Condition: \ \frac{dZ}{dP} = 0$$

Optimal Solution

Setting the first derivative to zero:

$$C^U \Pr(D - P^* > 0) - C^O \Pr(P^* - D > 0) = 0$$

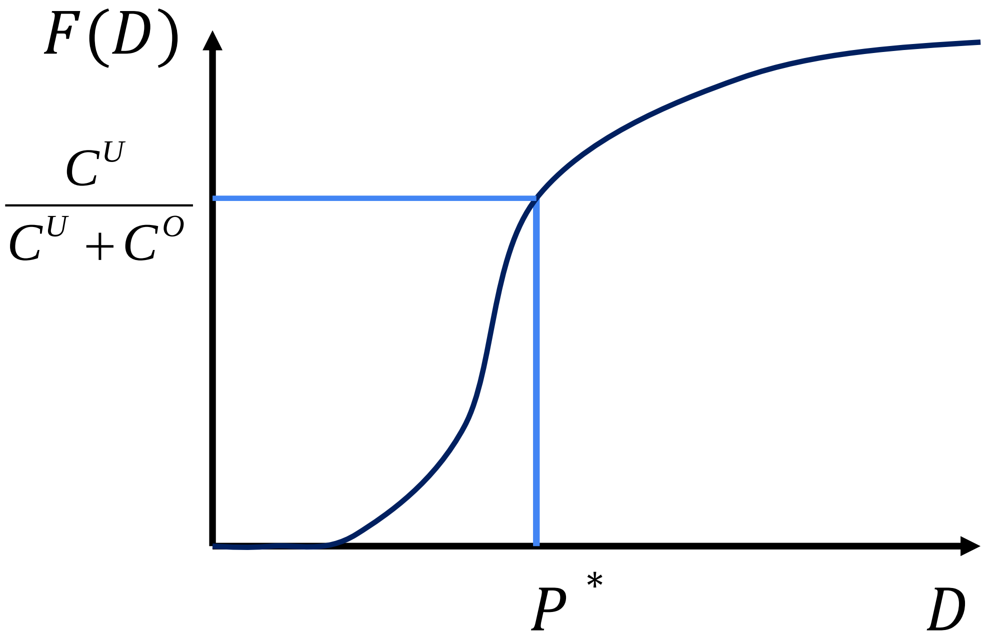
and letting $F(P) = \Pr(D \le P)$ denote the cumulative distribution function....

$$C^U(1 - F(P^*)) - C^O F(P^*) = 0$$

and therefore....

$$F(P^*) = \frac{C^U}{C^U + C^O} \longrightarrow P^* = F^{-1}\left(\frac{C^U}{C^U + C^O}\right)$$

Finding P*



Closing Remarks

- Operations Research (OR) bridges mathematics and other fields with societally important problems
- Career opportunities are endless because OR applies to almost any organization and field
- Demand for people with OR skills is high and growing fast

Putting Theory into Practice

How to Connect with Me and IOE

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Slides are on my website:

Search: @umichioe

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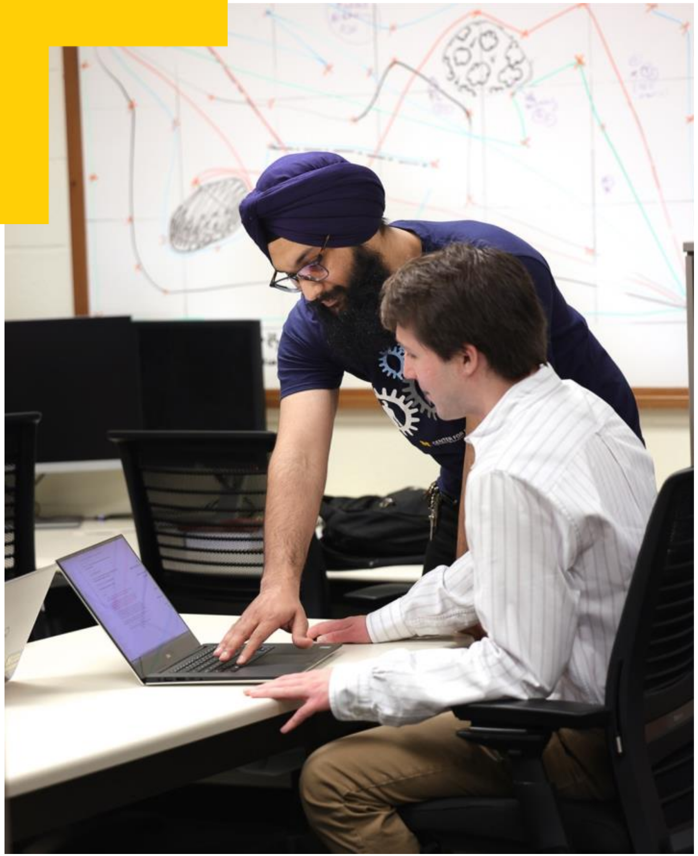
ioe.engin.umich.edu

Be a Solution

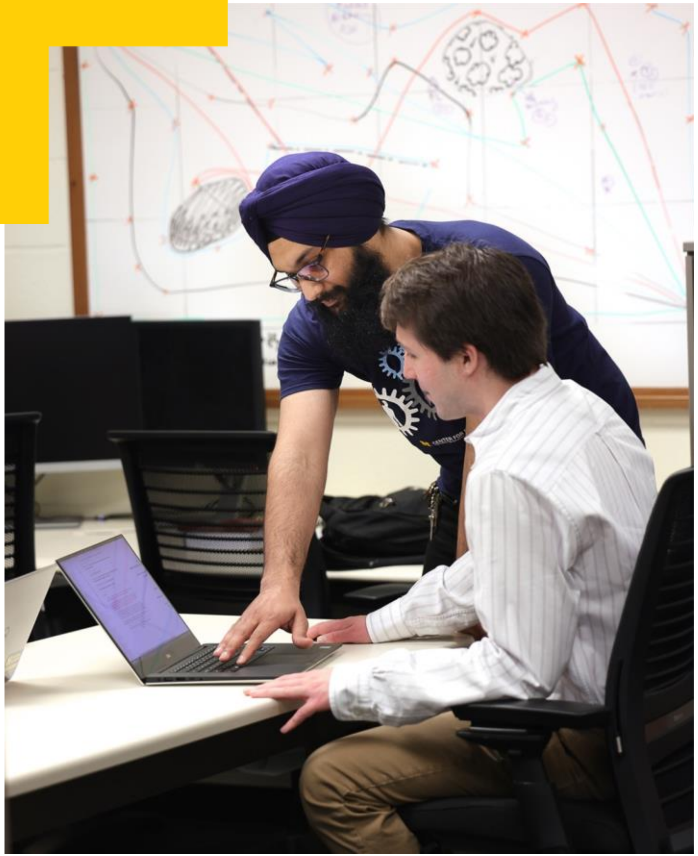
INDUSTRIAL & OPERATIONS ENGINEERING
UNIVERSITY OF MICHIGAN

Creating People-First Solutions



Why Pursue Your PhD in IOE at U-M?

- Outstanding faculty and staff
- Our top ranking attracts recruiters from all schools and industries
- Guaranteed funding
- Benefits such as healthcare and dental
- Fellowships for the first year



Why Pursue Your PhD in IOE at U-M?

- Flexibility to create a customized plan of study
- A wide breadth of opportunities across a top-ranked university
- Access to more than 60 graduate student certificate programs

Department Highlights

30 IOE faculty and 30+ faculty affiliates

Weekly speakers from academia and industry

Mentored teaching opportunities

Department Highlights

Vibrant community, social activities, and student orgs

Peer support and professional development

IOE Faculty Research

Theory and Methods:

Data Analytics

Human Systems Integration

Optimization

Stochastic Systems

IOE Faculty Research

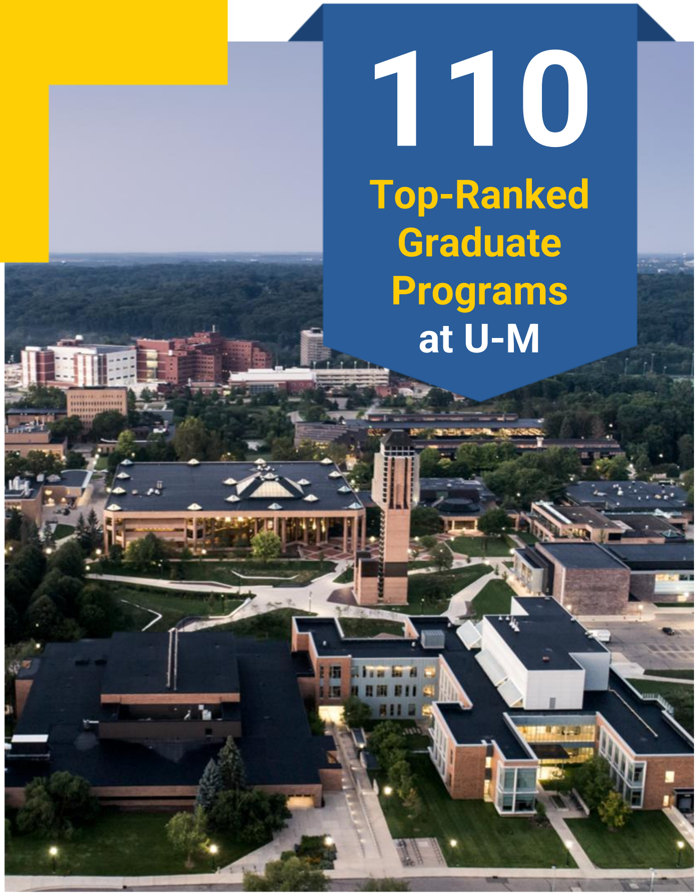
Industry Applications:

Business Operations

Health and Human Safety

Transportation and Mobility

Energy and Sustainability



Extensive Interdisciplinary Opportunities

U-M IOE PhD candidates can take classes and work with faculty from different engineering departments, Michigan Medicine, and other top-ranked U-M schools and colleges such as the Stephen M. Ross School of Business, the School of Information and the School of Public Health.

Business Operations and Analytics

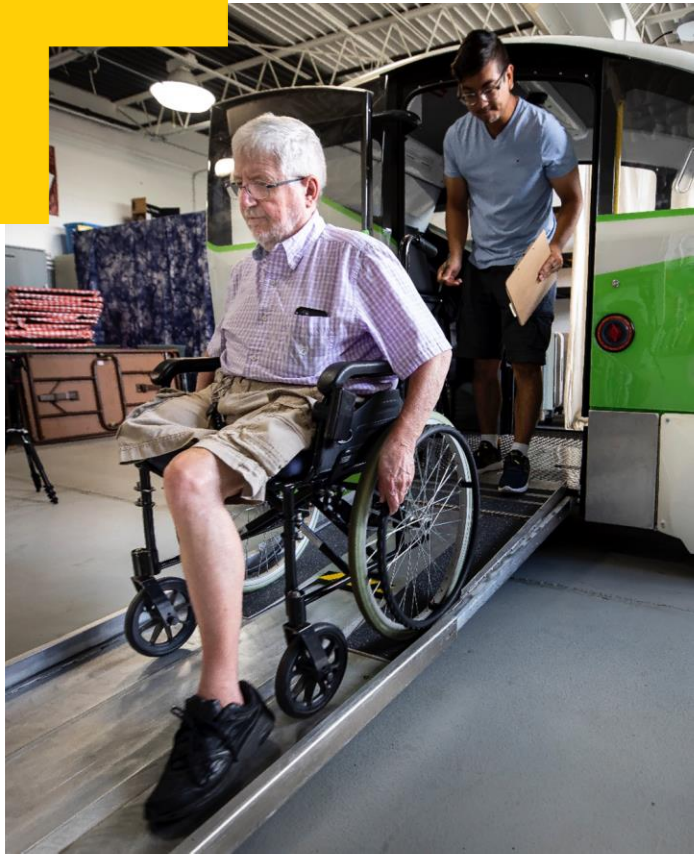
All sectors of the modern economy rely on fundamental quantitative tools for analysis, prediction, and optimization, to leverage data for the purposes of improving decision making.

Energy and Sustainability

Energy utilization and sustainability are linked through their influence on communities, climate change, and economics.

Health and Human Safety

Health and human safety involve many important decisions that affect people's lives, including the management of expensive resources in health systems, complex clinical treatment decisions, and the design of safe environments for people to live and work.



Mobility and Transportation Networks

Mobility and transportation systems research uses data-driven analytics, human-centered design principles, computer simulation models, and experimental studies to design and utilize advanced technologies.

Methodologies

IOE faculty and students create new tools and methods to extend the frontier of what is possible in our field using methodologies including:

- Data Analytics
- Human Systems Integration
- Optimization
- Stochastic Systems

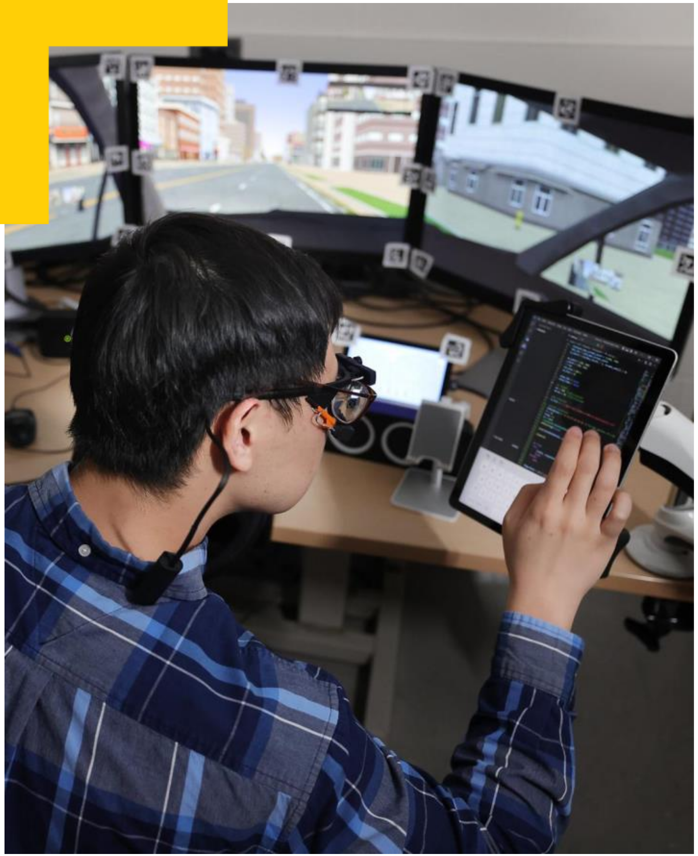
Data Analytics

Data science research uses principles from computation, machine learning, statistics, and mathematics to develop methods to analyze data and gain insight and knowledge about underlying systems to improve decision making.

Data Analytics

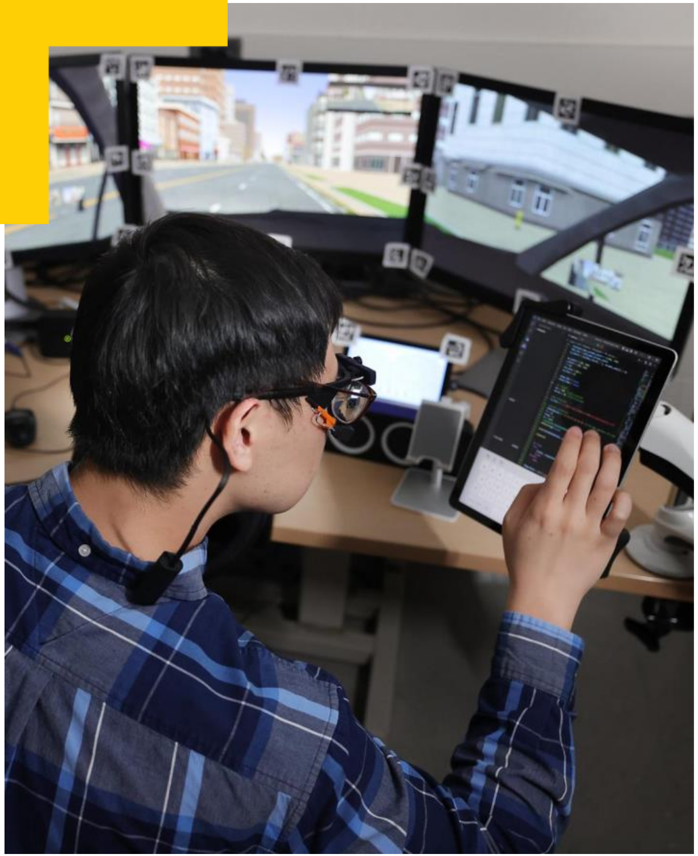
This area includes:

- Big Data Analytics
- Predictive Analytics
- Adaptive Learning



Human Systems Integration

This multidisciplinary research considers how human cognitive and physical needs and capabilities should be integrated into the design and development of modern sociotechnical systems to improve their safety and performance.



Human Systems Integration

This area includes:

- Human-Automation Systems in Transport Operations
- Wearable Sensors and Technologies
- Inclusive Design

Optimization

Methodological research in optimization uses techniques of algebra, geometry, analysis, computation, and combinatorics to develop and analyze algorithms for fundamental optimization models that can be applied broadly.

Optimization

This area includes:

- Integer Optimization
- Robust and Stochastic Optimization
- Combinatorial Optimization and Approximation Algorithm
- Continuous Optimization

Stochastic Systems

Stochastic systems are represented by stochastic processes that arise in many contexts (e.g., stock prices, patient flows in hospitals, warehouse inventory/stocking processes, and many others).

Stochastic Systems

This area includes:

- Queueing Systems
- Markov Decision Processes
- Reliability and Maintainability

Our Ph.D. graduates work all over the world in a wide variety of areas and roles:

Academics

Engineering, Business, Data Analytics, and Mathematics departments at top universities.

Our IOE graduates work all over the world in a wide variety of areas and roles:

Government

U.S. Air Force, Army, Los Alamos National Laboratory, National Laboratories, NASA, U.S. Navy, RAND, Sandia, and more.

Our IOE graduates work all over the world in a wide variety of areas and roles:

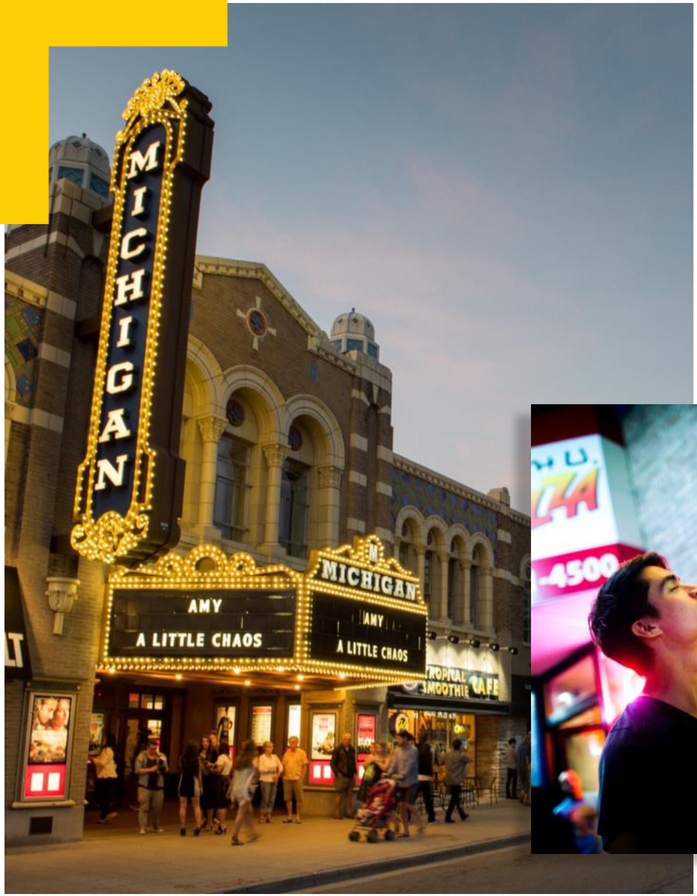
Industry

Amazon, Apple, Delta Airlines, Ford, General Motors, Google, IBM, Mayo Clinic, McKinsey & Co., Microsoft, Meta, Pratt & Whitney, etc.

Student Organizations

Institute for Operations Research & Management Sciences (INFORMS)

Serves the scientific and professional needs of OR/MS students, educators, scientists, investigators, and practitioners.



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